CSC 463 - Winter 2019
Problem Set 1, due at the start of tutorial on Friday January 25

Each problem set counts for 10% of your mark, and it is important to do your own work. You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. Copying assignments is a serious academic offense and will be dealt with accordingly.

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(1) Give a Turing machine which copies its input string, after a single blank. That is, if the input is a string w over the alphabet $\{0, 1\}$, then the machine should halt with tape configuration $w \textvisiblespace w$. Present your Turing machine either in the style of the course notes "Turing Machines and Reductions", page 3, or as a state diagram like that in the text. Explain in English how your TM works.

Note: This is mostly a practice exercise and will count very little.

For the following problems: You may use variants on Turing machines including enumerators as described in Section 3.2 of the text.

(2) A *Turing machine with left reset* is similar to an ordinary Turing machine, but the transition function has the form

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{\mathrm{R}, \mathrm{RESET}\}.$$

If $\delta(q, a) = (q', a', \mathrm{RESET})$, when the machine is in state q reading an a, the machine's head jumps to the lefthand end of the tape after it writes a' on the tape and enters state q'. Show every decidable language is accepted by a Turing machine with left reset which halts on all inputs.

(3) Prove that a language $A \subseteq \Sigma^*$ is semi-decidable if and only if there is a decidable binary relation $R \subseteq \Sigma^* \times \Sigma^*$ such that for all $x \in \Sigma^*$

$$x \in A \iff \exists y \; R(x, y).$$

(4) Show that every infinite semi-decidable language has an infinite decidable subset.