CSC 463 - Winter 2019
Problem Set 3, due at the start of tutorial on Friday March 15

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(1) Recall that an undirected graph $G = (V, E)$ is 3-colorable iff there is a map $f : V \to \{\text{red, blue yellow}\}$ such that no edge is assigned the same color to both its end points.

Define **3COL** $= \{\langle G \rangle \mid G \text{ is a 3-colorable graph}\}$.

Give an explicit reduction showing **3COL** $\leq_p$ **3SAT**.

(2) Let us say that the map f defined in Problem 1 is a *partial* 3-coloring of $G = (V, E)$ if the domain of f is a subset $V' \subseteq V$.

Define the NP problem **Partial-3COL** as follows:

Instance: $\langle G, f \rangle$ where f is a partial 3-coloring of the graph G.

Question: Can f be extended to a three-coloring of G?

Give an explicit reduction showing that **Partial-3COL** $\leq_p$ **3COL**.

(3) Consider the following decision problem:

Nice-SAT

Instance: $\langle \varphi \rangle$, where φ is a CNF formula such that every clause either consists entirely of unnegated variables or entirely of negated variables.

Question: Is φ satisfiable?

Show that **Nice-SAT** is NP-complete. (You may use the fact that **3SAT** is NP-complete.)