CSC 463 - Winter 2019
Problem Set 4, due at the start of lecture on Monday April 1

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(1) A *tautology* is a Boolean formula which evaluates to true (i.e. 1) under all truth assignments. Let TAUT be the language $\{\langle\varphi\rangle : \varphi \text{ is a tautology}\}$.

Recall that coNP $= \{A \subseteq \Sigma^* \mid \overline{A} \in \text{NP}\}$.

(a) Show that TAUT is in coNP.

(b) Show that if TAUT is in NP then NP = coNP.

(2) Let EXP $= \bigcup_k \text{TIME}(2^{n^k})$ and NEXP $= \bigcup_k \text{NTIME}(2^{n^k})$ be the classes of languages decidable by deterministic (respectively, nondeterministic) Turing machines with running time $O(2^{n^k})$ for some constant k.

Both P $\stackrel{?}{=}$ NP and EXP $\stackrel{?}{=}$ NEXP are open questions. However, it is known that if P = NP, then EXP = NEXP. Prove this fact!

Hint: For a language $A \in \text{NTIME}(2^{n^k})$, consider the "padded" language

$$A' = \{x1^{2^{|x|^k}} : x \in A\}$$

where $x1^{2^{|x|^k}}$ is the string formed by x followed by $2^{|x|^k}$ many 1's.

(3) Let 2SAT be like 3SAT except that the given formula has exactly two literals (involving two distinct variables) per clause. The purpose of this question is to show that 2SAT is NL-complete.

Recall that a literal has one of the forms $p, \overline{p}$, where p is a variable. If ℓ is $\overline{p}$, then $\overline{\ell}$ is p.

Given a 2CNF formula φ we associate a directed graph $G_\varphi = (V, E)$, where V is the set of all literals ℓ such that either ℓ or $\overline{\ell}$ occurs in φ, and for every clause $(\ell_1 \vee \ell_2)$ in φ we put the directed edges $(\overline{\ell_1}, \ell_2)$ and $(\overline{\ell_2}, \ell_1)$ in E.

(The idea is that if a truth assignment τ satisfies the clause $(\ell_1 \vee \ell_2)$, then if τ makes ℓ_1 false then ℓ_2 must be true, and if τ makes ℓ_2 false then ℓ_1 must be true.)

(a) Show that given any literals ℓ_1 and ℓ_2, if there is a directed path from ℓ_1 to ℓ_2 in G_φ then there is a directed path from $\overline{\ell_2}$ to $\overline{\ell_1}$, and every truth assignment to φ which satisfies φ and ℓ_1 also satisfies ℓ_2.

(b) Use part (a) to show that φ is unsatisfiable iff G_φ has a directed cycle which includes both p and $\overline{p}$, for some variable p.

(c) Use part (b) to show that 2SAT is NL-complete. (Use the fact that PATH is NL-complete.)