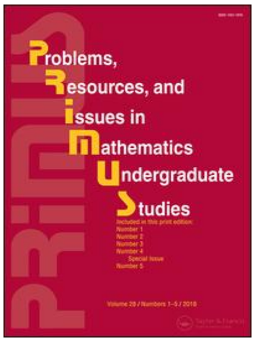

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Using Peer-Assisted Reflection in Math to Foster Critical Thinking and Communication Skills

Susanna Calkins, Sharisse Grannan and Jason Siefken

Abstract: This study explores the impact of Peer-Assisted Reflection (PAR), a structured active learning strategy that emphasizes peer feedback and reflection, on students' perceptions of mathematical thinking, and of the roles their peers and their instructors play in their learning process. This study also examines the impact of PAR on the students' ability to evaluate mathematical arguments and communicate those evaluations in writing, which has not been specifically measured in prior research on PAR. The findings suggest that the PAR intervention not only increases students' ability to communicate effectively, but also gives them a newfound recognition of the importance of developing communication skills in mathematics. Additionally, students' thinking about mathematics shifts as they come to value the exploration of multiple perspectives in solving math problems. Many students explicitly note their increased appreciation for the role of their peers in the learning process.

Keywords: Peer-assisted reflection, critical thinking, writing, communicating mathematics

1. INTRODUCTION

Over the last two decades there has been increased emphasis on including writing, reflection, communication, and revision as ways to enhance critical thinking in first-year math courses [11]. Indeed, in 2004, the Mathematical Association of America's Committee on the Undergraduate Program in Mathematics issued a report [4], for STEM in general, and mathematics in particular, in which they recommended

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that all courses be designed in such a way that learning could be assessed through the acquisition of important skills. Those skills included "analytical, critical reasoning, problem-solving, and communication skills" as well as the development of crucial "mathematical habits of mind." This report was written in response to the perception that many math classes in higher education, as commonly designed and implemented, do not substantially support one or more of these essential skills. Moreover, most undergraduates have never learned how to explicitly communicate their mathematical thinking to others, a common point of frustration for many faculty who teach math.

More specifically, this same report [4] also stressed that educators should seek to assess students' ability to:

> (i) state problems carefully, modify problems when necessary to make them tractable, articulate assumptions, appreciate the value of precise definition, reason logically to conclusions, and interpret results intelligently; (ii) problem solve with a willingness to try multiple approaches, persist in the face of difficulties, assess the correctness of solutions, explore examples, pose questions, and devise and test conjectures; and (iii) read mathematics with understanding and communicate mathematical ideas with clarity and coherence through writing and speaking.

These ideas are echoed in the broader context of critical thinking, which can be defined as the process by which students "grapple with complex issues, consider multiple perspectives, question authoritative sources, and develop [their] own nuanced interpretation" [15]. Critical thinking is also viewed as the ability to to push oneself out of "conceptual ruts" [3, 7], and to be more purposefully contemplative, or critically reflective, of one's own learning [13].

As communication skills have become more strongly linked with critical thinking and analytical skills, educators have increasingly sought to build reflection, peer feedback, and group learning opportunities into their curriculum [6], using a range of collaborative peer learning strategies. Drawing on several decades of research, David Boud and colleagues [1] note that peer learning collectively strives to promote several key learning outcomes. These outcomes include a sense of responsibility for one's own learning, and the learning of others; practice working with others who may have very different perspectives and ideas; practice asking questions, as well as answering and reflecting on answers (an ability that may be more readily fostered among one's peers, when the instructor is not present); the willingness and ability to more deeply communicate challenges and issues related to the subject matter with a peer. As Boud explains, "[Students] are able to articulate what they understand and to be more open to be critiqued by peers, as well as learning from listening to and critiquing others." (p. 8) Moreover, peer learning offers students the ability to communicate their ideas around content, as well as the ability to defend or examine their understanding when challenged. Peer learning also requires students to manage their own learning, demonstrating that "learning to cooperate with others to reach mutual goals is a prerequisite for operating in a complex society." (p. 9) Finally, peer learning offers "opportunities for giving and receiving feedback on one's work and a context for comparing oneself to others," a process ultimately at the heart of lifelong learning and improvement.

Collaborative peer learning strategies include Inquiry-Based Learning, in which students construct knowledge for themselves and with their peers by exploring mathematical questions and explaining their ideas [8], Team-Based Learning, in which students learn from their peers in rich communal interactions [6], and Peer Instruction, which allows students to reflect individually and then "instruct" one another about their responses [5]. In this study, we focus on a peer learning strategy called Peer-Assisted Reflection (PAR).

1.1. PAR

PAR is a pedagogical strategy developed by Daniel Reinholz [12] to enhance students' conceptual understanding, problem solving, and communication skills in introductory calculus courses. With PAR, students: (i) individually work on a difficult problem outside of class; (ii) self-reflect; (iii) exchange feedback with a peer in class; and (iv) revise their work outside of class before handing it in. It is designed, in part, to help students "transition from external feedback to self-monitoring." In his study, Reinholz implemented PAR in several introductory calculus sections, with all students completing PAR questions, but with only treatment sections engaging in the peer reflection process. He notes a significant reduction in the Drop, Fail, Withdraw (DFW) rate of introductory calculus sections that used PAR, as well as an increase in scores on every type of examination question, including purely computational questions. Moreover, when comparing the written homework and interview of PAR and non-PAR sections, he found that PAR "help[ed] students develop the perseverance required to solve challenging problems" [11]. He concluded that both the willingness to persevere and the stronger performance was enabled by the peer-reflection process of PAR, rather than just the process of solving hard problems.

Initially developed for calculus, the PAR process was adapted by the lead author for use in linear algebra, focusing more intentionally on developing students' awareness of their own role in supporting the learning of their peers and developing their critical thinking skills by requiring students to evaluate one another's arguments. This paper focuses on: (i) the impact of PAR on students' attitudes about mathematical thinking and their own role in learning in a linear algebra class; and (ii) the impact of PAR on students' abilities to provide critically reflective and constructive feedback to peers by evaluating mathematical arguments at a meta-level (rather than just communicating their thoughts to peers). We also include reflections on adapting PAR to a new course and training Teaching Assistants (TAs) to facilitate PAR.

2. STUDY OVERVIEW

We conducted a two-part study: Part 1 examined the impact of PAR on students' perceptions and attitudes towards their own learning and mathematical thinking; and Part 2 considered the impact of PAR on students' evaluation and communication skills. Specifically, we were driven by these questions: What impact does PAR have on students' perceptions of mathematical thinking; their perceptions of the roles of their peers, their instructor, and themselves in the process of learning; their ability to communicate mathematical arguments; and their ability to evaluate mathematical arguments and communicate those evaluations in writing?

We systematically examined students' self-reported attitudes about the mathematics and the process of learning in a math context, and we gathered quantitative data on students' ability to critique and give written feedback on another student's work. Following the PAR intervention, students expressed a newfound recognition of the importance of communication in mathematics. Our findings suggest that the intervention also increased their ability to communicate effectively about their own mathematical thinking. Additionally, students' thinking about mathematics shifted as they came to value the exploration of multiple perspectives in solving math problems, an important skill for critical thinking [14]. Often these varied perspectives were discovered through peer discussion, with many students explicitly noting their increased appreciation for and awareness of the role their peers played in the learning process [1].

2.1. Study Context and Sample Populations

This study was carried out at a mid-sized research-intensive private university located in the Midwestern United States, in a collaboration between an instructor of mathematics (the lead author), an expert in

Table 1. Study context and sample populations

Study Questions	Samples	Methods	Analysis
Part 1. Impact on perceptions and attitudes over time			
What impact does PAR have on students' perceptions of mathematical thinking? What impact does PAR have on students' perceptions of the roles of their peers, their instructor, and themselves in the process of learning?	13 first-year students in an honors-level 3-course sequence that includes one course in linear algebra (*Intervention I*); 10 first and second-year students in a non-honors course on linear algebra (*Intervention II*)	Pre- and post-PAR written reflective assignments for *Intervention I* & *II*; Post-PAR focus group with *Intervention I* students	Comparison of written reflections before and after the PAR intervention; Focus group data was subjected to thematic analysis and considered in relationship to written reflection data
Part 2. Impact on evaluation and communication skills			
What impact does PAR have on students' ability to communicate mathematical arguments? What impact does PAR have on students' ability to evaluate mathematical arguments and communicate those evaluations in writing?	13 first-year students in an honors-level 3-course sequence that includes one course in linear algebra (*Intervention I*); 10 first-year students in a different honors-level 3-course sequence that includes one course in linear algebra (*Comparison I*)	Evaluation of Arguments assignments	Evaluation of Arguments assignments were scored using a rubric; *Comparison I* did not have the PAR intervention and served as a comparison group

learning and teaching, and an expert in assessment and evaluation. Undergraduates who attend this university tend to be highly-motivated and high-achieving. We evaluated the impact of PAR on two student populations taking linear algebra courses (*Intervention I* and *Intervention II*), with a third as a comparison group (*Comparison I*), by means of qualitative self-reflection assignments and quantitative assessment of student work (See Table 1).

Intervention I and *Intervention II* both used PAR when learning linear algebra. *Intervention I* consisted of 13 students from a year-long introductory honors math sequence taught by the lead author. *Intervention II* consisted of 10 second-year students enrolled in a one-term linear algebra course. *Comparison I* consisted of 10 students from two comparable year-long introductory honors math sequences taught by other instructors. *Intervention I* and *Comparison I* students were both enrolled in three-term math sequences, one term of which was focused on linear algebra. All courses had a one-hour-per-week lab section lead by a TA.

2.2. Implementation of PAR

The lead author developed a set of seven PAR problems that were incorporated into the curriculum of *Intervention I* and *Intervention II*, both of which he taught. He designed the questions in such a way that students were were required to provide explanations, the questions could not be answered by blindly applying an algorithm from class.

For each PAR assignment, students had to write a complete solution to a difficult conceptual problem. Students reflected upon their solution, prompted by metacognitive questions such as: "How confident do you feel about your solution?" "Did you explain why (not just what)?" "Did you consult definitions of the mathematical terms you used?" (See the Appendix for an example worksheet.)

The first draft was due in the weekly lab sections that were specifically devoted to PAR. The first 10 minutes of the section were spent discussing how to give good feedback. Students were then randomly paired and exchanged drafts. Over the next 10 minutes, each student read the draft and provided written feedback. In the final 10 minutes, students conferenced about the written feedback, explaining to their partner about what they could do to improve their write-up. A revised draft was turned in the following day and graded for correctness and communication.

This implementation of PAR differed from Reinholz's in a few ways: there was more time devoted to giving and getting feedback (20 minutes compared with 10 minutes), the content was different (linear algebra compared with calculus), and the framing was different. PAR was cast (and explained to the students) as an activity to increase their ability to evaluate their peers' arguments for quality and correctness of the communication. This goes a step further than Reinholz's framing of PAR as a tool to help with understanding and explaining mathematical concepts [11, 12].

2.3. Methodology

2.3.1. Assessing Students' Ability to Evaluate Arguments and Communicate Feedback

Students in *Intervention I* and *Intervention II*, all of whom had the PAR intervention, were given a pre-course and a post-course reflective homework assignment. The pre-course assignment asked students to write about: (i) the kind of thinking required in a math class; (ii) how this thinking compares to that of other disciplines; (iii) what they think it means to be right or wrong in mathematics; and (iv) how they view the roles of their peers, the instructor, and themselves in the process of learning. The post-course assignment asked students to reflect upon their learning process and whether or not their thinking had changed during the course. Students were given their original responses to the pre-course assignment for reference. Two of the authors analyzed pre-course and post-course reflections, subjecting the qualitative data to thematic analysis, developing a coding scheme based upon emergent themes [2]. The authors attained inter-rater reliability by coding the responses separately, checking codes against one another, and engaging in discussion until consensus was achieved. A comparative analysis revealed similarities and differences in the pre and post data [10]. The inclusion of *Intervention II* provided information about how non-honors students who participate in PAR may have shifts in perspectives about mathematical thinking.

In addition, *Intervention I* students participated in a 30-minute focus group conducted during the last week of the year. Conducting a focus group reveals group dynamics and enables researchers to "capture shared lived experiences" inaccessible through other methodologies [9]. The nine students who chose to participate were entered into a $25 gift card drawing immediately after the focus group. The students were asked to describe what it was like to participate in PAR (explaining their answers in writing, giving and receiving feedback with peers, and making revisions to their work). They discussed ways in which PAR supported their learning or detracted from their learning, as well as aspects of it that they felt were challenging or enjoyable. This enabled us to understand which impacts they specifically attributed to the PAR intervention.

2.3.2. Assessing Students' Ability to Evaluate Arguments and Communicate Feedback

The lead author designed Evaluation of Arguments assignments. These assignments consisted of writing samples taken from responses to Calculus I PAR problems. Students were then asked to give feedback to the authors of these writing samples in a manner similar to their PAR assignments.

We developed a rubric (see the Appendix) in order to measure specific critical thinking skills, focusing on the degree to which students could evaluate arguments by identifying both strengths and weaknesses. We included a scheme for scoring the identification of missing steps in the argument. Dwyer, Hogan, and Stewart [7] note that identifying an argument's omissions is part of the overall analysis of the argument's strength or weakness. Clarity of communication, a skill often undeveloped within math courses, was emphasized in the PAR assignments, and we included a score for this dimension within the rubric. Finally, we awarded points for correctness. The scoring was calibrated by a second faculty member in the mathematics department who scored a random subset of the Evaluation of Arguments responses using the same rubric.

Intervention I students were given an Evaluation of Arguments assignment three times throughout the year. The third and final Evaluation of Arguments assignment was administered electronically during the lab section. *Comparison I* also completed the third Evaluation of Arguments assignment electronically. *Comparison I* students participating in the study received a $10 incentive for responding to an online survey that included this assignment and some of the questions from the pre-course reflections assignment described below. We received a 43% response rate and randomly selected 10 of the responding students to serve as a comparison group, so that we could begin to understand the potential impact of the PAR intervention. However, this was not a controlled experimental design given the number of other potential intervening variables such as differing pedagogical styles, course content, and receiving incentives versus grades.

2.3.3. Study Limitation

As stated previously, our comparison group for the Evaluation of Arguments assignments was not a true control group, given that the courses were not identical. The main difference between them, in addition to the style of individual instructors, was the order of content (linear algebra in the first term for *Intervention I* and linear algebra in the last term for *Comparison I*) Furthermore, the nature of the *Intervention I* program (*Intervention I* students took math, physics, and chemistry together as a single cohort) means that *Intervention I* students formed tight bonds and might feel more comfortable working with and relying on each other

Table 2. Descriptions of thinking expected in a math class ($n = 20$)

Description of thinking used in a math class	*n*	Exemplary quote
Math requires logical/rational/ analytical thinking or is a deductive process	12 (60%)	"I expect to engage in reasoning and logical thinking in a math class."
math is a process of prob-lem-solving	8 (40%)	"The type of thinking most often associated with math is prob-lem solving."
math thinking involves the application of prior know-ledge or concept	8 (40%)	"A student learns a conceptual topic, internalizes that informa-tion, and then takes their learn-ing to a deeper level by applying overarching ideals to spe-cific problems."
math is a process of cre-ative thinking	6 (30%)	"… matching functions that could go together in sequences. This type of thinking is more used in 'creative' classes."
math is a process of making connections	6 (30%)	" I expect to be doing thinking that requires making connections spanning multiple concepts and ideas."
math is a process of linear thinking or following steps in a process	4 (20%)	"A logical progression of ideas should lead one to a final best answer."
math is abstract or conceptual (or a conceptual process)	4 (20%)	"I expect the thinking done in math class to be more conceptual and application-based than in most other classes."
math involves finding patterns	3 (15%)	"I expect to be doing structured thinking, such as computing numbers and finding patterns within equations and functions."
math is a process of crit-ical thinking	3 (15%)	"I expect critical thinking in a mathematics class."

*Percentages do not total to 100% because students' responses often included multiple themes.

than *Comparison I* students. Finally, the Evaluation of Arguments was administered to *Intervention I* students during class time and *Comparison I* completed it out of class, incentivised by a gift card. However, true control groups are difficult to achieve, and we believe that the similarities between the students, their courses, and the conditions provided for legitimate comparisons. The consistencies of responses to the questions about math thinking (shared below) testify to the strength of the comparison group model.

3. IMPACTS ON PERCEPTIONS AND ATTITUDES OVER TIME

Below we present our analysis of the qualitative data collected through the pre-course reflective assignment, post-course reflective assignment, and focus groups. Overall, students entered their math course with a fairly sophisticated view of mathematical thinking, but their perspectives became more nuanced at the end of the term, with a significant shift towards: (i) valuing peers in the learning process; (ii) recognizing the importance of multiple perspectives when solving problems; and (iii) valuing the importance of strong communication in mathematics.

3.1. Pre-course Reflective Assignment

When *Intervention I* and *Intervention II* students were asked to describe the kinds of thinking associated with taking a math class, their responses fell broadly into nine categories (see Table 2).

Students tended to identify math as a process of problem solving that involves logical thinking or deductive reasoning. Aspects of critical thinking, as outlined in Stein and Haynes [14] also came through in their responses. For example, over a third of the students (8/20) said that math involves the application of prior knowledge or concepts to new problems. Furthermore, some said that math was a process of making connections (6/20) and identifying patterns (3/20). Three students explicitly mentioned critical thinking. One student's description of mathematical thinking combined the concepts of creativity (another theme) and application: "[Math requires] creativity to apply concepts we already know in different ways or in combination with other concepts."

Additionally, over a third of students (8/20) said that the kind of thinking they do in math is similar to thinking they do in non-STEM disciplines. As one student explained:

> [Math requires] a very holistic and creative type of thinking that is often associated with subjects in the arts like philosophy or creative writing… the creativity to apply concepts we already know in different ways or in combination with other concepts.

Almost twice as many students (14/20) said the thinking they do in math is similar to what they do in other STEM disciplines. As one student noted, "the subjects that might be classified as similar in method are any that involve mathematics or statistics, such as physics, chemistry, engineering, etc." This same student also pointed out that this kind of thinking was different from thinking that occurs in

Table 3. Changes in perception

	Math thinking	Right or wrong answers in math	Roles of self, peers, instructors in learning
Stayed the same	3	5	2
Did not address	1	9	3
Changed	16	6	15
Total	20	20	20

non-math fields, such as in the humanities: "Classes that lack strong mathematical backing like literature and philosophy involve modes of thought that are wholly different."

Students were asked whether, in math, an answer is always right or wrong. The students were split, with almost half (9/20) agreeing with the binary perception of math. Most of those students gave an emphatic "yes" response, though a few had more nuanced responses, for example, explaining how there may be multiple ways of arriving at the single correct answer. Eight students gave "yes and no" responses, a few saying it depends upon the question type and others discussing the way one could arrive at the correct solution for the wrong reasons. Six students referenced human limitations or made epistemological arguments when explaining why not every answer to a math problem is either right or wrong. For a few, the evolving nature of the discipline of math means that math knowledge is limited.

3.2. Post-course Reflective Assignment

At the end of the course or course sequence, *Intervention I* and *Intervention II* students were asked to reflect on their pre-course reflective assignment and explain how their thoughts and opinions changed or stayed the same. In two areas, the kind of thinking done in mathematics and the roles of self, peers, and instructors in student learning, the majority of students described shifts in their perspectives. These changes were either changes in direction (e.g., "I used to think this, but now I think the opposite.") or they were described as reinforced perspectives (e.g., "I said this, but now I have new, very strong reasons to believe it"). In contrast, for the question of whether an answer is always either right or wrong, less than a third of students described a change in perspective; many did not address it at all in their reflections (See Table 3). On average, students changed their perspectives in two of these three areas; only one student did not describe any change of thinking in any of these specific areas.

A deeper qualitative analysis of the students' reflections revealed three specific shifts in students' perspectives. A single student's statement could indicate all three or none of these shifts. More than half described a new recognition of the importance of clear communication in math; more than half described a new understanding of the importance of multiple perspectives in math; and almost three-quarters described having gained a new appreciation for the role of peers in learning. These themes were echoed in the focus group data. Many students specifically attributed these shifts to the PAR assignments.

3.2.1. A New Appreciation for the Role of Peers in Learning

In reflecting upon their roles as learners, in relation to their peers and their instructors, three-quarters of the students indicated that their perspectives had changed since the beginning of the academic period. Some wrote about a changed view of the instructor–student relationship, where they now see themselves as more active in their learning and view the instructor as more of a "guide" or "facilitator." However, a newfound appreciation for the role of their peers in their learning was even more pronounced.

For example, one student explained:

> Over the past year, I've learned a LOT from my peers, and often had to rely on them to explain things to me in a way such that I can understand easily … Learning from peers happened very often … especially on certain questions that require a change of perspective.

The student also noted that there was a reciprocity to the peer assistance, adding:

> Hopefully I've been able to provide help for at least a few other people in the class as well, because I've definitely benefited from the others in this class.

For another student, the role of peer interactions in learning was something he or she had always valued, but by the end of the sequence had come to value to an even greater extent:

> I still agree that the role my peers play is one of a source of new ideas and inspiration when I get stuck, but my ideas have been enhanced by how incredible my peers have been this year. They've done so much more than just contribute ideas—they've supported me, inspired me … .

Even more to the point, as a third student explained: "Peers are more than competitors, they can be valuable teachers and useful audience members."

Of the 14 students who noted a new appreciation of the role that their peers can play in the learning process, six specifically attributed the change to PAR. As one explained:

> I now see the role of peers as being similarly important to that of the teacher in that [they can] help one another understand concepts. For example, this happened when working on the PAR.

Similarly, the importance of peers in learning also came up during the *Intervention I* focus group on PAR:

> Through your peers reading the PAR, you can improve the communication to make it not so ambiguous and it actually serves as a driving force for you to improve your communication skills.

3.2.2. A New Understanding of the Importance of Multiple Perspectives in Math

In addition to asking students generally about their perspectives on the kind of thinking done in math, we also asked them a more directed question at the end of their introductory assignment:

> Professors in the English department list "evaluate multiple perspectives," "reframe questions and issues," and "examine central issues and assumptions" as goals for their courses. Are these goals relevant to math?

Original responses to this question were fairly broad and unspecific. However, when students revisited their comments, many (55%) commented specifically on the benefits of exploring multiple perspectives. In the post-course reflection, one student equated thinking about problems from multiple perspectives with "critical thinking" and said that this helps students "understand the concepts on a level beyond just the basic equations and methods we learned to arrive at a solution." Another shared that "different perspectives help to reframe questions and issues and sometimes, if we are lucky, simplify them." One student, who wrote at the beginning of the course about the importance of having perspectives from different people noted that, "it's also useful to independently think of various ways to approach the same problem." One of the 11 students with a deepened appreciation for exploring multiple perspectives attributed this shift in thinking to the PAR assignments.

3.2.3. Increased Recognition of the Importance of Clear Communication in Math

Half the students explained in the post-course reflective assignment that they had gained a new appreciation for the importance of communication in math. This theme is particularly remarkable given that the topic was not asked about in either assignment. A couple of these students expressed surprise in how similar math was to other writing-focused courses, such as English. One student reflected:

> I did not previously stress the pertinence of effective communication with my audience; I assumed that it was sufficient if I had the correct answer to be able to fleetingly describe my process.

Another explained that for mathematical work to have usefulness beyond "the confines of your own mind," "being a good explainer is incredibly important."

Three of the 10 students who commented on this attributed the change to the PAR assignments, which, as one student described, "require explanation of every definition, theorem, and logical step included in the solution. … I've realized the importance of such practice."

When asked in the focus group specifically about PAR and any potential ways it may have added to or supported their learning, many *Intervention I* students indicated that it helped build their communication skills, and helped them recognize the importance of such skills. As one student said, the PAR assignments helped her move from being "really wordy and not clearly getting my points across" to, in the end, feeling "able to succinctly say what I wanted to say and explain it." Several focus group and survey participants noted that having to clarify one's thinking in words can deepen one's understanding of concepts. For example, one student shared, "It was helpful to get feedback on effective communication and ideas, which is also a very important part of learning and it helped clarify the concepts."

4. IMPACTS ON EVALUATION AND COMMUNICATION SKILLS

Below we present our quantitative analysis of the Evaluation of Arguments assignment. The Evaluation of Arguments assignments worked in the following way: students were given a PAR problem and a sample solution to this PAR problem. They were asked to give feedback to the author of this sample solution specifically addressing: (i) whether the logic was correct; (ii) the strengths and weaknesses of the answer; (iii) whether all steps were explained; and (iv) how the answer should be revised. While previous studies using PAR as an intervention have analyzed student responses to PAR questions, as far as the authors are aware, this is the first time the effect of PAR on a student's ability to give feedback has been directly analyzed.

4.1. Evaluating Arguments

In the Evaluation of Arguments assignment, *Intervention I* outperformed *Comparison I* in categories of correctness, identification of strengths, identification of weaknesses, and communication. Overall, *Intervention I* students received an average rubric score of 49% whereas *Comparison I* received an average rubric score of 41%.

The greatest discrepancy occurred in the category of communication. Every response was given points for how well the response was communicated. *Intervention I* and *Comparison I* scored a 78% and 60%, respectively, on communication. Furthermore, no *Intervention I* student scored zero for communication, whereas one third of *Comparison I* students scored zero points.

An example of a high-scoring and low-scoring response to the Evaluation of Arguments assignment is presented below:

High-scoring:

> 1) Their logic is correct, as using a Riemann sum is a valid way to estimate the area under a graph. 2) Their approach is valid, but they do not discuss how they drew the rectangles (left-endpoint, right-endpoint) or why they specifically used rectangles (instead of trapezoids for example) 3) The response is missing a final answer/conclusion, but more importantly, they are missing an explanation of why they chose the method they did and an analysis of the accuracy of the given method could be an appropriate addition to the explanation. 4) When revising their answer, the student should recount the boxes, and add an explanation of the accuracy of their method and why they chose it.

Low-scoring:

> Their logic is incorrect, because they are neglecting a large area under the curve that does not fit within a rectangle. They have more exact methods of calculating at their disposal—such as Riemann sums—and should be using one of those to avoid neglecting such a large area. It is also incredibly inconsistent, as they are rounding to the rectangle as they see fit and are not following a consistent method of finding the area. This is problematic, as their estimation could vary as a result of their opinion. This is why a Riemann sum would be better. If they further made their method consistent and then explained that well, they would essentially be doing a Riemann sum and their logic would be more correct. This is the path they should follow for revision. They should also double check their work and make sure their counting is correct and consistent. One strength of their argument is that they used smaller intervals, so that if their approximation was consistent, then it would be more exact.

Of note, both the high-scoring and low-scoring responses point out accuracy considerations, but only the high-scoring response directly communicates what should be done to improve the argument and its writeup.

Another strong difference between *Intervention I* and *Comparison I* scores was in the mathematical correctness of their feedback. A full 90% of responses given by *Intervention I* students had zero mathematically false statements, whereas only 60% of *Comparison I* student responses contained zero mathematically false statements.

5. INSTRUCTOR'S PERSPECTIVE

In previous iterations of the course, the lead author had attempted to build peer feedback into his teaching, but found that students often did not know how to provide constructive remarks to one another, and he did not know how to develop this skill. He also wanted to develop students' critical thinking and communication skills more purposefully, particularly since those were core learning objectives in his courses. After staff at the Learning and Teaching Center (LTC) brought PAR to his attention, he thought that PAR's structure could help him address some of the challenges with including formal writing in a math course, particularly scalability. PAR's selling points were that it provided structured guidance to students on how to give meaningful critical feedback on each other's writing and how to be reflective of their own writing.

Based on the description provided in [11], the lead author decided to try PAR. In personal lead, Reinholz shared Calculus I PAR questions. Though the structure of PAR could be transferred to a linear algebra course, these questions could not, and a whole new set of linear algebra PAR questions had to be created. These questions may be found at https://github.com/siefkenj/linearalgebra-PAR.

The remainder of this section will address challenges and observations encountered by the lead author when using PAR (for the first time) in a linear algebra course.

5.1. Considerations for Implementation

From the instructor's perspective, there are five main points to consider when implementing PAR in a linear algebra class: (i) developing expectations for mathematical writing; (ii) designing PAR questions; (iii) training TAs; (iv) training students on how to give good feedback; and (v) making PAR feel integrated with and important to the class.

Expectations of Mathematical Writing

At some universities linear algebra serves as an Introduction to Proofs course. That was not the case at the lead author's institution, which left a challenging question: what is good mathematical communication when the emphasis is not on proofs? The lead author struggled to get a clear picture of this, eventually settling on a fuzzy distinction between "proof" and "rigorous argument":

> Whereas a proof should follow the cultural traditions of mathematics (a pattern of lemma-theorem where every step is justified by referencing a theorem or axiom combined with specific uses of language, like "let" for introducing a variable, etc.), a rigorous argument does not need to match the style of a proof and in a rigorous argument, one can be more liberal with the distinction between a fact and a theorem (i.e., a justified fact).

With this working definition, a good rigorous argument is a few steps away from being a complete proof, but the emphasis is shifted to connecting ideas rather than logical exactness. The focus of PAR was to provide an answer to the question backed up by a rigorous argument.

Designing PAR Questions

PAR questions are hard to design. They need to be deep enough that there is something to write about, but if they are too hard, students will spend all their energy solving the question and not on the writing. In linear algebra this is particularly challenging because "easy" topics such as row-reduction and systems of equations are very hard to write about! (For example, the question "Explain why the solutions to a system of equations doesn't change after row reduction" is much harder than "Prove that an orthonormal set of vectors is a basis of some subspace.")

On several occasions, the lead author's questions proved too challenging. During office hours, students would spend the entire time solving the question and there was no time left to discuss their writing. Some TAs similarly observed that, during discussion time, many students were talking together about how to solve the question rather than giving feedback to the written work.

Training TAs

Different from Reinholz, our TAs conducted PAR during lab sections. This meant the TAs had to be trained in how to facilitate PAR as well as how to give feedback to students on their writing. Since there was a sufficient number of TAs interested in PAR, the lead author arranged a common training session. In this training session, TAs first did PAR (albeit at an accelerated pace) with a Calculus I PAR question. TAs role-played either as themselves or as a student, purposely inserting errors in their responses. They had 7 minutes to write a response. After trading papers, they spent 3 minutes giving silent feedback and another 5 minutes conferencing. After that, three samples of actual student PAR responses (to the same PAR question) were passed around and TAs, in pairs, worked to give feedback to the authors of those PAR response. This was followed with a group discussion on what would constitute helpful feedback.

Ideally, the lead author would have had resources available to follow up with TAs throughout the term, giving them feedback on how they facilitated the PAR process. Facilitating PAR while maintaining student engagement and having students believe that PAR is a worthwhile activity is very hard. Future implementers who have their TAs facilitate PAR should not underestimate the difficulty. Even experienced TAs may need support.

Training Students on Giving Feedback

Training students on how to give feedback to one another is critical to the success of PAR. Reinholz accomplished this with an exercise he called "darts." Throughout the term he posted one sentence PAR responses and asked students to judge whether they were "perfect", "close", or "off-the-mark" and then lead a classroom discussion on what feedback would be helpful to give to the author of each response. The lead author used a variant of this process: sample responses to a non-PAR question were provided and students were lead in a discussion, facilitated by the TA, about what feedback would be useful to the authors. This was done immediately before students exchanged papers.

The lead author modified the "darts" process for two reasons: (i) sample student responses to these PAR problems were not available (because the questions were brand new); and (ii) concern that students would not put effort into answering the PAR questions if they were going to see sample answers in lab. In retrospect, students had a hard time connecting the modified "darts" with how they should give feedback to their peers. In future iterations, the instructor will base the "darts" exercise off of the same question in the PAR assignment, to better motivate students and allow them to apply skills from one task to the next.

Integrating PAR within the Course

In the lead author's implementation, PAR was used in conjunction with traditional homework, and, though PAR did not take a significant amount of class time, it did take a big chunk of students' work time. A PAR draft along with traditional homework was typically due on a Thursday, and the PAR final was due on Friday. As a result, many students focused on their traditional homework and came to lab with a substandard PAR draft.

Separate from due-date issues, having PAR take place solely in lab left some students feeling like PAR was something their TA cared about but was not something their instructor cared about. Having some PAR sessions run by the instructor might mitigate this feeling.

6. CONCLUSION

We conducted a mixed-methods study to understand the impact of PAR; not simply on test scores, but on the critical thinking skills we suspected it supported. We found that it positively impacted students' ability to evaluate mathematical arguments and communicate those evaluations in writing. The findings suggest that PAR additionally gave students a new-found recognition of the importance of communication skills in mathematics and exploring multiple perspectives.

6.1. Recommendations

Although we believe PAR can be a meaningful strategy that instructors might use to engage students, enhance communication and evaluation skills, and ultimately improve learning, the process is not without challenges. For example, creating conceptually-rich and measurable PAR problems as well as effectively integrating PAR into the course was difficult. Although the lead author discussed PAR with his students during class, all PAR conferencing and feedback-training was carried out in lab by TAs. Because he did not implement PAR himself, he thought that the PAR activities came across as disconnected from the rest of the course activities.

Given the potential that PAR offers in terms of student learning and engagement, we offer here some strategies and recommendations that other instructors might find useful. First, instructors can seek to create an environment that encourages inquiry and collaboration, and one in which it is appropriate and good to take risks and make mistakes. In order to complete PAR, students have to believe that they can do the problems, but also be willing to accept feedback and use it to improve. To this end, students must be trained in PAR, particularly in the process of giving and receiving effective feedback. If a TA administers PAR in a lab section, conducting feedback training with the primary instructor during class time might make PAR feel more integrated. Facilitators of PAR, whether an instructor or TA, must be sufficiently trained in PAR and understand the underlying goals and purpose. Lastly, we strongly recommend that the instructor carefully thinks through the PAR assignment schedule. If regular homework is assigned in addition to PAR, the final PAR draft should be due at the same time. Otherwise, students will put off their PAR problems and their initial drafts may be sub-standard.

We believe the PAR process is flexible enough to be implemented within multiple mathematics disciplines, as demonstrated by our adaptation of PAR to linear algebra. However, when implementing PAR in a new course or at a new university, instructors should be prepared to adapt and iterate. Instructors might have to develop multiple PAR questions before landing on one with the right balance of challenge, complexity, and ease. And, instructors might have to tweak how they do TA training and student training (and even instructor training) for PAR. The effort required to design the questions and to train students, TAs, and instructors, is rewarded by students' growth in critical thinking skills and their valuing of those skills.

6.2. Final Reflections

PAR was an effective tool for improving students' communication as well as their attitudes about math as a collaborative process both in an honors and a non-honors course. Originally designed to increase students' mathematical understanding in calculus, we extended the goal of PAR to include developing the skill of evaluating others' arguments and their own written communication. In addition, we learned that students participating in PAR began to recognize the value of specific critical thinking skills, including identifying multiple perspectives and communicating effectively. They also experienced an increased appreciation for the role they play in their peers' learning.

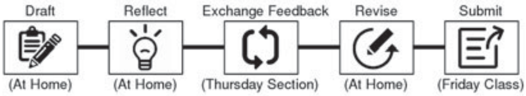
Ultimately, although PAR is a highly-structured pedagogical strategy requiring a great deal of time, practice, and care to implement, we found the process allowed students to meaningfully reflect on their own learning, enhance their communication skills, and contribute to the learning of their peers.

APPENDIX A: PAR EXAMPLE

Math 240: Peer-Assisted Reflection #1

Due Dates: 9/29 (draft), 9/30 (final) Name: ____________________

The PAR Process



Note: Write your draft on a separate sheet and bring it to section. I will pass out a hard copy of this sheet for your self-reflection; bring that to section as well. Turn in both on Friday, plus your final draft.

Problem Statement

The newly-appointed queen of a newly-discovered land hires three explorers to map her territory: Emily, Jack, and Lila. The explorers have their own equipment and their own quirks.

Jack Jack has a miscalibrated compass—it is off by 45°. When Jack thinks he's walking east, he is actually walking due northeast. When Jack walks in what he thinks is a cardinal direction, he measures distance accurately.

Emily Emily is a careful explorer with an accurate compass. When Emily measures distance in a cardinal direction, it too is accurate.

Lila Lila is an excitable explorer. Her compass is accurate, but when she walks north, she skips and twirls. As a consequence, when Lila walks north, the distance she thinks she travels is *half* the distance she actually traveled.

Further, each explorer only walks in what they think are cardinal directions (i.e., north, east, south, and west). The queen declares that her palace is the center of the nation and that all measurements be made relative to her palace. She then sends the explorers on their way.

1. Emily finds the ruins of an ancient civilization located 70 miles east and 40 miles north of the queen's palace. She records the *royal coordinates* $(70, 40)$ in her logbook. The other explorers find the same ruins. What coordinates do they record in their logbook?
2. Each explorer is familiar with the Pythagorean theorem, and uses it with his/her own coordinates to compute the distance from the palace to the ruins. What distances do they compute? Are they all the same? Explain why any differences or similarities appear in the distance calculation.
3. The queen values accuracy and will behead anyone who reports inaccurate distances (strangely, the queen is just fine with inaccurate directions). Lila, unfortunately, cannot be convinced to change her coordinates. However, she might be convinced to use a different formula for distance, rather than the Pythagorean theorem. Is there a formula that Lila can use to accurately compute distances from her coordinates?
4. Suppose that Jack starts skipping and twirling and records a distance *half* as far as he actually travels when he thinks he's walking north. Is there a formula that he can use to compute distances accurately? How does this formula relate to Lila's? Explain.

Reflection

Turn the page and check off the icons that you think you did well; circle icons that you want feedback on.

APPENDIX B: EVALUATION OF ARGUMENTS

MATH 281-3 **May 18, 2017**
EVALUATION OF ARGUMENTS (*May 18, 2017*) **Name:**

Please take a few minutes to review the following problem. After you've reviewed it, see the two responses (a and b) which were generated from the work of former Calculus students. Please give feedback to each student, communicating the following:

(1) whether their logic is correct/mostly correct or incorrect/mostly incorrect;

(2) the strengths and weaknesses of their answer (if there are any);

(3) whether the response is missing steps and/or could be better explained; and

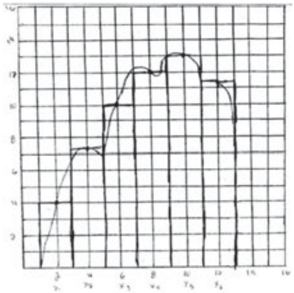
(4) specifically how the student should revise this answer.

Problem: In this problem, you will trace the shape of your hand and then estimate the area of that shape. On graph paper, first trace the shape of your hand. Next, devise a method to estimate the area of that shape. Explain in detail how this method works, but don't perform any numerical calculations.

1

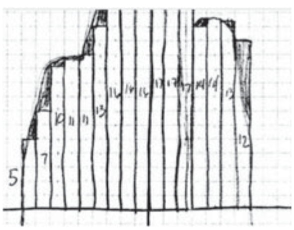
MATH 281-3 **May 18, 2017**

EVALUATION OF ARGUMENTS (*May 18, 2017*)



(a)

For my method, I used the midpoint Riemann summation. The midpoint estimation will have a closer approximation than the left-endpoint or right-endpoint approximation. This is because the midpoint estimation is larger than the left-endpoint value and smaller than the right, making its value between them.



(b)

The method I used was to add boxes to determine the area under the curve based on intervals of the rectangles. This will help approximate the area because we know the formula for the area of a rectangle. We then add up all the rectangle areas.

2

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BIOGRAPHICAL SKETCHES

Susanna Calkins, Ph.D., directs the Faculty Initiatives at the Searle Center for Advancing Learning and Teaching at Northwestern University. She is co-author of *Learning and Teaching in Higher Education: The Reflective Professional* and has authored/co-authored over 25 research articles related to learning and teaching, mentoring, and critical thinking in higher education. She also teaches in the

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Jason Siefken, Ph.D., is an assistant professor in the Department of Mathematics at the University of Toronto. He specializes in dynamical systems and aperiodic tilings and is interested in active teaching strategies and building students' mathematical writing skills. He has developed materials, projects, and writing assignments for multivariable calculus, differential equations, and linear algebra.