The Mathematical Experience

Creativity Theme Stream

Quest University Canada Block 1, Spring 2015

> *Mathematics is one of the oldest of sciences; it is also one of the most active, for its strength is the vigour of perpetual youth.* – Andrew Russ Forsyth

> *Mathematics is one of the few subjects that a student can study throughout high school and even few years into college without coming into contact with any results invented since 1800.* – Alan Hammond

This course will blend an exploration of important mathematical ideas with an examination of the place of math within the intellectual landscape. We will make conjectures and construct proofs, working with problems from fields such as topology and group theory. As we do mathematics, we will study works of both non-fiction and fiction and discuss issues such as the portrayal of math and its practitioners in the media, the nature of mathematical truth, women and members of underrepresented minorities in math, and genetic influences on mathematical achievement.

Central topics for the theme stream include the role of creativity in math, the nature of mathematical creativity, how one can experience or develop mathematical creativity, and implications for education.

By the end of this course you should be able to:

1. Understand key ideas from three areas of modern mathematics: number theory, group theory, and topology
2. Recognize important issues in the contemporary mathematics community
3. Communicate mathematics with greater clarity and more confidence levels
4. Create mathematics, and evaluate mathematical writing

Course Information

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Course Topics

We will explore important ideas in the following areas of modern mathematical research.

- Number Theory: Why is number theory called the 'Queen of Mathematics'? How do we describe the *poetry* of numbers? What are numbers made of? What qualities may make a number special?

- Group Theory: What is symmetry? How many wallpaper patterns are possible? How do we describe mathematical patterns? What qualities do different systems share?
- Topology: What is the shape of our universe? How do we envision ourselves in 4 dimensions? In what way is a donut the same as a coffee cup? How many different surfaces are there?

We will also examine the experience of learning, discovering, inventing, and critiquing new mathematics in the twenty-first century.

- What is mathematics, really?
- What is mathematical creativity?
- Who creates new mathematics?

Course Texts

The following books are available from the university bookstore. With the exception of *The Mathematical Experience*, all are mandatory. You will need a *paper* copy of each.

- Davis, Philip J., and Hersh, Reuben. *The Mathematical Experience*. New York: Mariner Books, 1999.
- Ogilvy, C. Stanley, and Anderson, John T. *Excursions in Number Theory*. New York: Dover Publications, 1988.
- Weeks, Jeffrey R. *The Shape of Space*. 2nd ed. New York: Chapman & Hall, 2001.
- Henrion, Claudia. *Women in Mathematics: The addition of difference*. Bloomington: Indiana University Press, 1997.
- Ogawa, Yoko. *The Housekeeper and the Professor*. New York: Picador, 2009.

Additional readings will be posted on the course website.

Weekly Meeting

I will schedule a meeting with each of you on each Thursday of the course to discuss your progress in the class. Please bring your journal, current work, and recently returned assignments with you.

Course Requirements

> *If I have made any valuable discoveries, it has been owing more to patient attention than to any other talents* - Sir Isaac Newton

To pass this course you will need to complete the following.

- 3 Mathematical Synthesis Assignments

- Mathematical Creations Project
- Book Review
- Mathematics in the Arts Poster
- Satisfactory preparation for and participation in a Panel Discussion
- Daily readings and journal entries

Synthesis Assignments

> *You have to be confused before you can reach a new level of understanding anything.*
> – Dudley Herschbach

Synthesis Assignments will give you the opportunity to contemplate, struggle with, and conquer mathematical problems. The questions on these problem sets are challenging and will probably look quite different from those that you have encountered in previous mathematics classes. The process of working through problems in this course mimics mathematical discovery, and working through them will help you to understand the modern mathematical experience.

These assignments will also allow you to develop your mathematical writing skills. Writing clear mathematics is not only important in math classes: it will help to improve your critical thinking and your ability to communicate ideas from other fields clearly. I have high expectations for your writing; please read the handout 'How to Write Mathematics' carefully before submitting your first Synthesis Assignment. If you have questions about what I expect, please come and speak with me.

You will complete one synthesis assignment on each of our mathematical themes: number theory, topology, and group theory.

Mathematical Creations Project

> *At first I'm terribly confused, but after a while I chip away at my wrong ideas until I'm left with an answer. So I think that I'm working in the sculptor mode, rather than the inspired painter.*
> *Inspiration starts things, but only hard work really gets anywhere.* – Bradley Efron

Over the course of the semester, you will work with a partner to create your own original mathematics. This project will give you the opportunity to more deeply understand what mathematical creativity is.

Over the first few days of the course, you will will work with your partner to think of a new concept from geometry based upon the concepts that you are already familiar with. You should do your best to come up with an original concept that you would like to learn more about. For example, concepts like scalene triangles or concentric circles are *not* original.

The following will be due on Thursday, January 8.

- A descriptive name of your creation (3 points)
- A definition of the concept (8 points)

  - Definitions in mathematics are very precise, and good definitions are difficult to write. Make sure that your definition is neither too broad nor too narrow.

- At least one prototypical illustration to accompany your concept (2 points)
- A log of your daily progress in designing and refining your concept (5 points)

Details on the main part of this project will be given at at the end of Week 1.

Book Review

> *As we will see, 'The Calculus Wars' does not meet any of these expectations. Moreover, with its frequent misspellings, its many sentences that would not pass muster in a high school writing class, and its abundance of typographical errors, 'The Calculus Wars' falls short of a reader's most basic requirements.* –Brian E. Blank

There are many wonderful books about mathematics available that read more like novels than textbooks. They can introduce you to new mathematics and to the process of creating new mathematics, by telling fascinating stories from the recent and long-ago history of this subject.

In this course, you will be reading a book of your choice about mathematics written for a general audience. You will then write a review for prospective readers. The purpose of a book review is to give prospective readers a preview of what the book is like, whether or not the reviewer enjoyed it, and details of where they can learn more. Writing this review will give you the opportunity to learn more about an area of mathematics that you are interested in, and introduce you to different styles of mathematical writing.

More details about this project, including possible book selections, are given on the course website. The final Book Review will be due on Friday, January 16.

Mathematics in the Arts Poster

> *The universe is written in the language of mathematics.* –Galileo Galilei

While our focus throughout this course will be on creativity within mathematics, people often experience mathematics through the arts. To give you the opportunity to study the deep connection between mathematics and the arts, you will create a poster that celebrates mathematics in a particular area of the arts. You will display your final poster on the last day of the course.

More details for this project are given on the course website. Notice that, prior to the final deadline, a general topic for the project is due on Friday, January 9 and that a more detailed description of the poster is due on Tuesday, January 20.

The potential topics for this poster are practically limitless. Possible ideas include: MC Escher's hyperbolic tessellations, the geometry of cubism, origami, symmetry in quilting, perspective drawing, the mathematics of magic, and mathematical structures in poetry. See the detailed description of this project for more ideas.

Panel Discussion

If I have seen further it is because I have stood on the shoulders of Giants.– Isaac Newton

A panel discussion is a moderated discussion between selected speakers on a specific topic, in front of an audience. On Friday, January 23 we will be staging six short panel discussions between groups of contemporary mathematicians. The purpose of these panel discussions will be to learn more about the lives of contemporary mathematicians and what it is like to do mathematics in the modern era.

You will organize the panels as a class during Week 2, and play the role of two mathematicians in two separate panels on the day of the discussions. More details on this project will be given at the beginning of Week 2.

Preparation and the Mathematics Journal

Obviously you work like hell and once in a while you notice something really unexpected. –David Donoho

The reading assignments will help you to understand mathematical material and will prepare you for class discussions. These assignments come in two flavours. Mathematical reading discusses the subject of mathematics and comes from the fields of number theory, topology, and group theory. You should aim to *understand* when you read mathematics and will often need to stop and think about what you read. See the next section for some reasons on why it is important to read mathematics outside of class. Second, you will be reading *about* mathematics and mathematicians. If you see references to mathematics in this type of reading, do not spend too much effort trying to understand the details of the arguments. I am more interested in the mathematical experience that these readings describe.

Mathematical Reading and Journal

The ability to understand mathematical texts is an important skill for *any* future mathematical study. This skill is vital for at least three reasons.

1. **Future Learning**. When you need to learn a mathematical concept on your own, your main resources will be written.

2. **Efficiency**. In an ideal world we might try to discover all the mathematics by ourselves, but this would be impractical. The great abundance of mathematical writing available allows us to learn from the experts.

3. **Learning to Communicate**. Just as reading many stories makes you a better storyteller, reading a lot of mathematics makes you a better mathematical communicator.

You will write about mathematics in your Journal daily. Each daily entry consists of two parts.

1. A brief summary of key points and reflections from the math reading, phrased in your own words. This should be more than simply a series of quotes or headings from the

reading; sometimes a shorter entry is better than a longer entry! It should include the following organizational features:

- Date
- Reference to the reading at the start of the entry (e.g. book and page numbers or article title)
- Page number references throughout the entry
- A summary of the key points from the reading.

2. Responses to assigned questions, written in a clear and easy to follow manner. It should include the following organizational features.

- Date
- If the question comes from the text, the question number
- If the question is posted online, a restatement of the question
- The work that you needed to get to your conclusion
- A clear conclusion

The Math Journal will serve several purposes.

- Writing a summary of what you read about helps you to synthesize new information.
- The record that you create will serve as a reference days or weeks after you first encounter the material.
- Completing daily problems helps you to gauge your understanding of new material and prepares you for the more difficult questions in Synthesis Assignments.
- Writing about mathematics for yourself helps to improve your mathematical communication skills in general.

I will be collecting the journals unannounced throughout the course, so please bring your journal to class daily.

Participation

- **Actively participate in class discussions,** asking questions, offering comments, and listening carefully to what others say. If you have a question about something, please ask! There will likely be other people who have the same question.
- **Be respectful of other class members and maintain a collaborative environment.** Contributing to a class discussion does not mean talking a lot. You should listen carefully to others' ideas and be careful about offering a critique. When you do object to others' ideas, be kind.
- **Respect course policies.**
- **Note Taking.** I recommend that you use a binder to organize your notes, as there will be frequent in-class handouts and worksheets that would be difficult to corral into a notebook.

Academic Integrity

While googling a homework problem or trading solutions with a classmate may seem like good strategies for doing well in this class, these actions will prevent you from learning material, refining your problem-solving skills, and developing self-sufficiency and self-esteem.

The consequences for cheating are severe. *Any* blatant academic dishonesty will result in failure of the course and immediate reporting to the Chief Academic Officer.

The following actions are *not* considered cheating.

- Discussing questions from Synthesis Assignments with classmates, building off of each others' ideas
- Using online resources to help you understand the content of the course or practice problems (e.g. problems that you do not submit)

The following actions *are* considered cheating.

- Looking for solutions to Synthesis Assignment problems online (e.g. by searching or posting on a message board).
- Copying the writing or explanations of mathematical work from someone else
- Using others' words or ideas without properly citing them

These examples are not comprehensive; if you have questions about whether something is considered cheating, please speak with me first.

Grading

Your final grade will be calculated as follows.

30%	Synthesis Assignments Each of the Synthesis Assignments will be worth 10% of your final grade
15%	Mathematical Creations Project
10%	Book Review
10%	Mathematics in the Arts Poster
10%	Panel Discussion
15%	Mathematics Journal
10%	Preparation for Class

The course grading scale is:

A	93-100%	B	83-86%	C	73-76%
A-	90-92%	B-	80-82%	C-	70-72%
B+	87-89%	C+	77-79%	D	60-69%

Narrative Evaluation

Any student at Quest can request a narrative evaluation (e.g. a written paragraph) in addition to their letter grade in any course. A narrative evaluation will give you more comprehensive feedback that you can learn from and additional information to present employers and graduate schools. If you wish to take advantage of this option, you have until the end of the 6th day of a course to sign up on the Registrar's Office Portal site.

Disability Accommodations

If you have a disability for which you seek accommodation, please make sure to have registered with the Learning Commons, as specified in the Student Accommodation Policy (`http://www.questu.ca/pdfs/_uploads/content/student_accommodation_policy.pdf`), and provide us with your Memorandum by the second day of class.

Additional Course Policies

- Please be on time to class. If you arrive for class and the door is closed this means that you are late. You will be permitted to be late 2 times without penalty during the block. On the third time, I will deduct 1% off of your final grade, and 1% more for each time that you are late.
- I expect that you will attend every class session. If you must miss a class for a valid reason (such as illness or a family emergency), please let me know *before* class. I reserve the right to ask for documentation to support your absence. For every class that you miss without a valid reason, 5% will be deducted from your *full* course grade.
- Bring pencils, paper, a scientific calculator, and a laptop to every class. You may also need to bring special materials such as scissors or pencil crayons to some classes, but will be given advanced notice when this is required.
- You cell phones must be off during class, and your laptops should be shut unless we are using them for a class activity.
- Always be respectful in your speaking and actions. Do not use profanity.
- All homework is due at the beginning of class, and all deadlines are absolutely firm. I will not accept late homework since we need to be able to discuss solutions in class, and because staying on top of deadlines encourages you to keep up with course material.
- If you need an extension on an assignment, I *must* see documentation, and you must place your request at least 24 hours ahead of the due date.
- Office hours: If you are not able to make my drop-in office hours, or need to speak with me privately, please e-mail me to arrange an appointment.
- E-mail: During the block I check my e-mail on weekdays at the beginning and end of the day, and sporadically at other times. Please do not e-mail me with questions that may be easily answered by looking at this syllabus, the course website, or asking other members of the class. Be polite and use proper English grammar.
- Please do not bring food into the classroom. You may bring drinks.

Reading Assignments

The anticipated reading assignments for the class are below. There may be changes in the reading (particularly in the mathematics reading) based on our progress in-class. The Day x readings are *due* on Day x.

Day 1

- *A Mathematician's Lament: How school cheats us out of our most fascinating and imaginative art form* by Paul Lockhart

Day 2

- Chapters 1 and 3 of *Excursions in Number Theory*
- *But Aren't Truth and Beauty Supposed to be Enough?*, James Gleick
- *How to Fall in Love with Math*, Manil Suri
- 'The Virtues of Mathematics' from *Dude, Can you Count?* by Christian Constanda (pp 277 –285)

Day 3

- Chapter 8 and Chapter 4 (pp 39–45) *Excursions in Number Theory*
- *Are giftedness and creativity synonyms in math?*, Bharath Sriraman

Day 4

- Chapter 4 (pp 45–54) and Chapter 7 of *Excursions in Number Theory*
- 'Beauty and Truth in Mathematics' by Doris Schattschneider from *Mathematics and the Aesthetic*
- *Equations are Art Inside a Mathematician's Brain*, Claudia Moskowitz

Day 5

- Chapters 11 (pp 135-139) and 2 of *Excursions in Number Theory*
- *Computers still can't do beautiful mathematics*, Gina Kolata
- 'Proof' from *The Mathematical Experience* by Philip Davis, Reuben Hersh, and Elena Anne Marchisotto

Day 6

- Begin *The Housekeeper and the Professor* (to be completed by Day 10)

Day 7

- Begin biographies for Panel Discussions

Day 8

- Sections 2, 3, 4, 7, 8, 10, 19, 20, 29 of *A Mathematician's Apology*, G. H. Hardy
- *The Math Gene: A Ticket to Wealth or Nerdiness?*, Ronald Lipsman
- *Impossible Cookware and Other Triumphs of the Penrose Tile*, Patchen Barss

Day 9

- Chapter 1 from *Women in Mathematics*
- *Massively collaborative mathematics*, Tim Gowers and Michael Nielsen
- *Classifying Frieze Patterns*, Sarah-Marie Belcastro and Thomas C. Hull

Day 10

- *The Housekeeper and the Professor*
- Watch *Flatland*
- Get started on reading for Day 11

Day 11

- Chapters 1 and 2 of *The Shape of Space*
- Download Torus Games on your computer (`http://www.geometrygames.org/TorusGames/`)
- Chapters 2 and 5 of *Women in Mathematics*
- *Why are there still so few women in science?*, Eileen Pollack

Day 12

- *How math got its Nobel*, Michael Barnay
- *Is there a Curse of the Field's Medal?*, Janos Kollar
- Chapter 3 of *Women in Mathematics*
- Chapters 3 and 4 of *The Shape of Space*

Day 13

- Chapters 5 and 6 of *The Shape of Space*
- 'Mathematics as an addiction: Following mathematics to its end' from *Loving and Hating Mathematics* by Reuben Hersch and Vera John-Steiner
- *The Mathematical Mind* by Ayala Ochert
- *If you think you're a genius, you're crazy*, Dean Simonton

Day 14

- Biographies for Panel Discussion

- Chapters 7, 13, and 14 of *The Shape of Space*

Day 15

- Chapter 15 of *The Shape of Space*
- Chapter 6 *Women in Mathematics*
- *Useful invention or absolute truth?*, George Johnson

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	Day 1 5 Patterns in Numbers The Primes What is Math?	*Day 2* 6 Patterns in the Primes Why Math? **Selection for Book Review Due**	*Day 3* 7 Changing Perspective: Modular Arithmetic What is mathematical creativity?	*Day 4* 8 Perfect, Lucky, and Fibonacci Numbers Mathematics & The Aesthetic **Part 1 of Mathematical Creations Project Due**	*Day 5* 9 Pythagorean Triples Diophantine Equations How does math grow? What is truth in math? **Math in Art Poster: Deadline for Direction**	
	Day 6 12 What is it like to do math research? **Number Theory Synthesis Due**	*Day 7* 13 Panel Organization **Panel Organization Forms Due**	*Community Day* 14	*Day 8* 15 The Set of Symmetries Symmetries as Actions The Ideal Mathematician	*Day 9* 16 Combining Symmetries Cayley Tables The Mathematical Community **Book Review Due**	
	Day 10 19 What is a Group? A Fictional Mathematician: *The Housekeeper and the Professor* **Group Theory Synthesis Due**	*Day 11* 20 Flatland Gluing to make new shapes Topology vs. Geometry Women and Minorities in Math **Outline of Mathematics in Art Poster Due**	*Day 12* 21 Nonorientable Surfaces The 3-Torus Mathematical Prizes Mathematics as a 'Young Man's Game'	*Day 13* 22 3-Manifolds New Manifolds from Old Manifolds Mathematical Minds: Creative Genius & Insanity	*Day 14* 23 Geometry of Surfaces Contemporary Mathematicians: A Panel Discussion **Panel Discussion Preparation and Notes Due**	
	Day 15 26 The Shape of Space Is Mathematics Discovered or Invented? **Topology Synthesis Due**	*Day 16* 27 Mathematical Creativity, Revisited Mathematical Creation Presentations **Mathematical Creation Project Due**	*Day 17* 28 Course Wrap-Up Mathematics in the Arts **Mathematics in the Arts Poster Due**			