Welcome to MAT137!

(Section L5101, Tuesday 6-9 and Thursday 6-9)

- Hi, I am Sourav
- My e-mail: ssarkar@math.toronto.edu (Please include MAT137 in the subject line)
- My Office Hours: Mondays, 5:00-7:00
- You will find all the relevant Zoom links on Quercus
- The webpage for the first half of this section is http://www.math.toronto.edu/ssarkar/1372020.html. This will tell you which videos to watch and also have the slides uploaded a few hours before class.
- Everything else (problem sets, tutorials...) you will find on Quercus. Regularly check Quercus and your emails for any update on the course.
- Read the course syllabus (all of it!). Some recent changes made.

Things to do

- Join Piazza, our online help forum.
- Review precalculus. http://uoft.me/precalc
- Check your emails regularly. If you have not gotten an email from me yet, there is a problem with your utoronto email address.
- Register in Gradescope
- Enrol in a tutorial!
- **For next day's lecture, watch videos 1.7 through 1.15.**
- **Today's topics: sets, quantifiers, logical statements and their negation.** (Videos 1.1 – 1.6)

Videos and textbook and problems

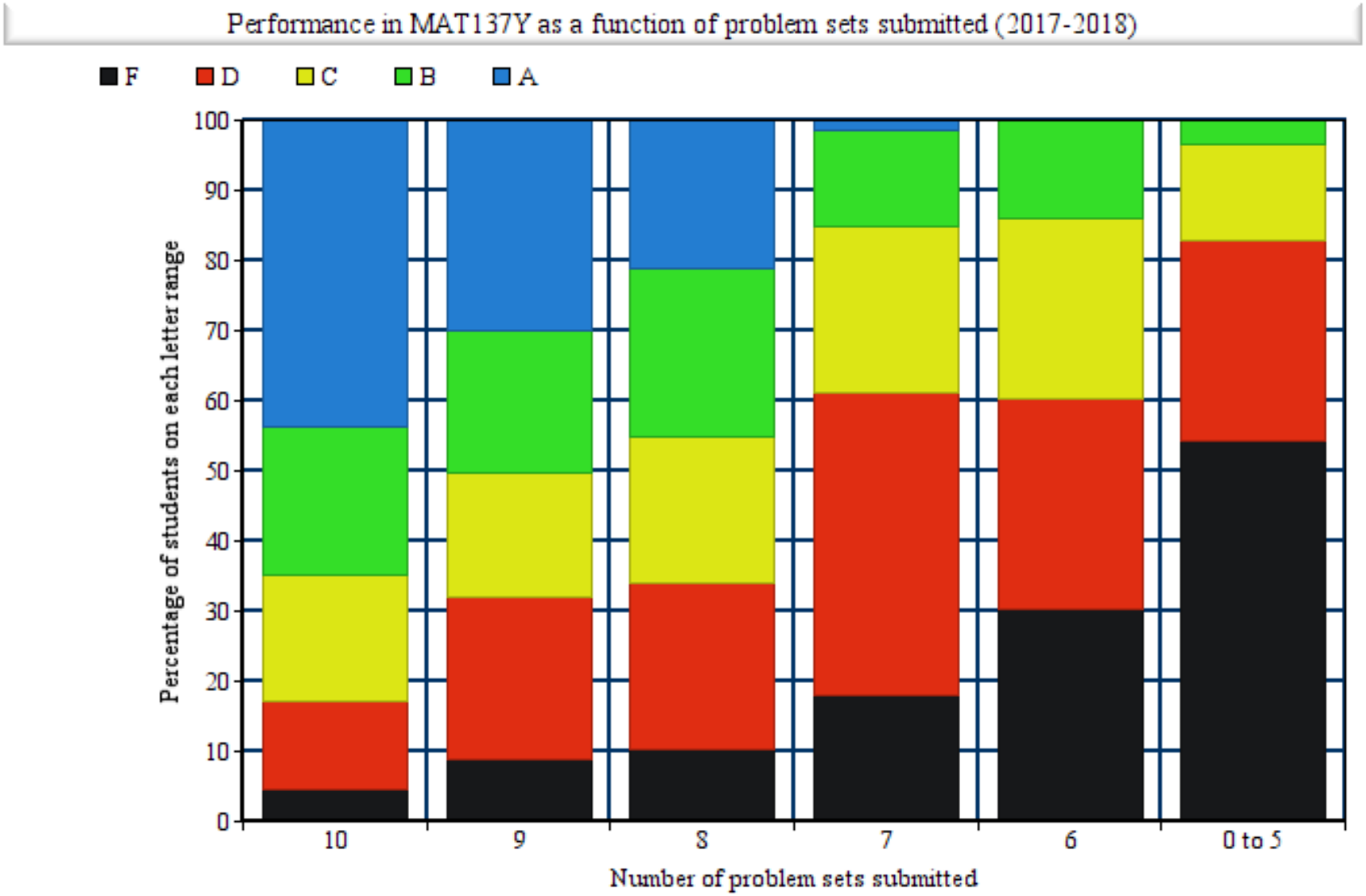
- The videos are your primary source and must be viewed before class. You can do all your learning with them.
- The textbook is not required.
- **Problem set 1** has been posted on Quercus. It is due on Wednesday, May 13th. (You need to submit the solutions on Gradescope)

What grades do students get in MAT137?

It depends on how many problem sets they submitted.

How did students do in past years?

MAT137 results from 2017–2018:



Course structure

- The class is in inverted format. It is very important that you watch the assigned videos before class.
- The main objective of the course is for you to understand the relevant materials, develop mathematical rigor, and problem solving abilities.
- So focus on understanding the concepts and not on memorization.
- This is a calculus class. But first, this is a logic and critical thinking class.
- This is going to be a tough course! Be prepared to work hard!
- Don't be afraid/shy to ask ("silly"!!) questions or be wrong in class. No one is here to judge you. And you shouldn't be here to judge others either. **Certainly, I will not!**

How to work?

- Open today's slides alongside Zoom.
- Take notes and solve problems like in-person class.
- Listen and Watch. No distractions!
- Mute your mic and camera to avoid lag.

How to participate?

- Do the activities as we go
- Reply to Polls!
- Use the chat if you have any question/doubt and when you give an answer
- Discuss in Breakout rooms. Socialize with your peers in the rooms, discuss math (use the audio/video).
- Ask questions, answer them

Phew!! That was too much of lecturing!! :(

Let's get started!!

Topics: Sets and notation, basic quantifiers, negations

Sets

Describe the following sets in the simplest terms you can.

1. $[2, 4] \cup (3, 10)$
2. $[2, 4] \cap (3, 10)$
3. $(\pi, 3)$
4. $[7, 7]$
5. $(7, 7)$
6. $A = \{ x \in \mathbb{R} : x^2 < 7 \}$
7. $B = \{ x \in \mathbb{Z} : x^2 < 7 \}$
8. $C = \{ x \in \mathbb{N} : x^2 < 7 \}$

Sets defined with quantifiers

Problem 1. Describe the following sets in the simplest terms you can.

1. $A = \{ x \in \mathbb{R} : \forall y \in [5, 7], x < y \}$.
2. $B = \{ x \in \mathbb{R} : \exists y \in [5, 7] \text{ such that } x < y \}$
3. $C = \{ x \in [5, 7] : \forall y \in [5, 7], x < y \}$.
4. $D = \{ x \in [5, 7] : \exists y \in [5, 7] \text{ such that } x < y \}$
5. $E = \{ x \in [5, 7] : \exists y \in \mathbb{R} \text{ such that } x < y \}$
6. $F = \{ x \in [5, 7] : y \in \mathbb{R}, x < y \}$

Set difference

Given two sets A and B, we define

- $A \setminus B = \{x \in A \ : \ x \notin B\}$. This set is called "A minus B".

What are the following sets?

- $[0, 1] \setminus (0.5, \infty)$
- $\mathbb{R} \setminus [0, 1]$
- $[0, 1] \setminus \mathbb{R}$

Raise your hand if...

- A:= {Students whose name starts with A,E, I, O or U}
- B:={Male students}

Raise your hand if you are in $(A \setminus B) \cup (B \setminus A)$. This set is called the symmetric difference set of A and B (written as $A \Delta B$).

Even integers

Let E be the set of even integers. Write E in set-building notation.

Even integers

Which of the following is a correct description of the set E of even integers?

1. $E = \{ n \in \mathbb{Z} : \forall a \in Z,\ n = 2a \}$
2. $E = \{ n \in \mathbb{Z} : \exists a \in Z \text{ such that } n = 2a \}$

Rational numbers

Let $\mathbb{Q}$ be the set of rational numbers. Write $\mathbb{Q}$ in set-building notation.

Order of quantifiers matters! A lot!

The following two statements are identical *except* for the order of the two quantifiers:

$$\forall x \in \mathbb{R},\ \exists y \in \mathbb{R} \text{ such that } x < y.$$

$$\exists y \in \mathbb{R} \text{ such that } \forall x \in \mathbb{R},\ x < y$$

Try to phrase each of these statements as simple English sentences. Notice that their meanings are **very** different!

Which is correct?

The following two statements are identical *except* for the order of the two quantifiers:

$$(1) \qquad \forall x \in \mathbb{R},\ \exists y \in \mathbb{R} \text{ such that } x = y.$$

$$(2) \qquad \exists y \in \mathbb{R} \text{ such that } \forall x \in \mathbb{R},\ x = y$$

Prove your claim!

Mother

Let

$$H = \{ \text{ humans } \}$$

True or False?

1. $\forall x \in H$, $\exists y \in H$ such that y gave birth to x
2. $\exists y \in H$ such that $\forall x \in H$, y gave birth to x

What is negation?

The negation of a statement is a statement which is false whenever the original statement is true and true whenver the original is false.
What is the negation of the statement "every student in this classroom likes Mathematics"?

Negation

Write the negation of these statements as simply as possible:

1. Every student at U of T has a cellphone.
2. There is a country in the European Union with fewer than 1000 inhabitants.
3. I like pistachios and walnuts.

Elephants and vacuous truths!

True or False?

1. There is a pink elephant in this room.

2. All elephants in this room are pink.

Negation 2

Write the negation of this statement without using any negative words ("no", "not", "none", etc.):

> *"Every page in this book contains at least one word whose first and last letters both come alphabetically before M."*

Hint: Identify all the quantifiers. Negate each clause and move systematically.