Welcome back to MAT137!

(Section L5101, Tuesday 6-9 and Thursday 6-9)

- Please fill out the course evaluation form. You can access the evaluation form via the 'Course Evals' tab in Quercus or by clicking on the personalized link that you received automatically by e-mail through the course evaluation system
- Recording of previous lecture uploaded on Quercus.
- **For next day's lecture, watch videos 6.11-6.15.**
- **Today's topics: Optimization, Indeterminate Forms, L'Hopital's Rule** (Videos 6.1-6.10)

Let's get started!!

Any question from previous class?

Intervals of monotonicity

Let $g(x) = x^3(x^2 - 4)^{1/3}$.

Find out on which intervals this function is increasing or decreasing.
Using that information, sketch its graph.

To save time, here is the first derivative:

$$g'(x) = \frac{x^2(11x^2 - 36)}{3(x^2 - 4)^{2/3}}$$

Inequalities

Prove that, for every $x \in \mathbb{R}$

$$e^x \geq 1 + x$$

Hint: When is the function $f(x) = e^x - 1 - x$ increasing or decreasing?

Proving difficult identities (Do as an exercise)

Prove that, for every $x \geq 0$,

$$\arcsin \frac{1-x}{1+x} + 2\arctan\sqrt{x} = \frac{\pi}{2}$$

Hint: Take derivatives.

The classic farmer problem

A farmer has 300 m of fencing and wants to fence off a rectangular field and add an extra fence that divides the rectangular area in two equal parts down the middle.

What is the largest area that the field can have?

Distance

Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.

Fire

You hear a scream. You turn around and you see Donald Trump is on fire. Literally. Suppose you decide to help him out and not let him burn! Luckily, you are next to a river.

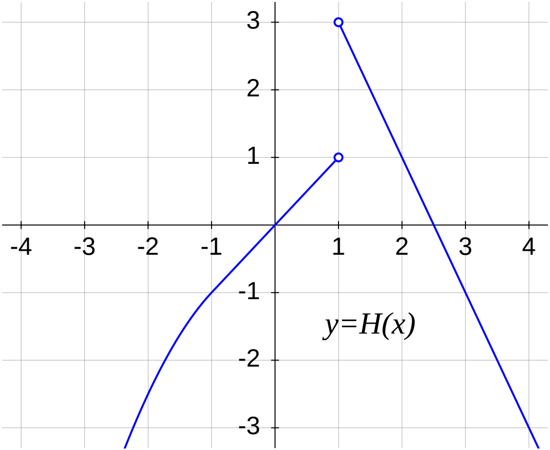
DT is 10 meters away from the river and you are 5 meters away from the point P on the river closest to DT. You are carrying an empty bucket. You can run twice as fast with an empty bucket as you can run with a full bucket.

How far from the point P should you fill your bucket in order to get to DT with a bucket full of water as fast as possible?

Warm up: Limits from graphs

Compute

$$\lim_{x \to 0} \frac{H(x)}{H(2+3x)-1}$$



Proving something is an indeterminate form

1. Prove that $\forall c \in \mathbb{R}$, $\exists a \in \mathbb{R}$ and functions f and g s.t.

$$\lim_{x \to a} f(x) = 0, \quad \lim_{x \to a} g(x) = 0, \quad \lim_{x \to a} \frac{f(x)}{g(x)} = c$$

   This shows that $\frac{0}{0}$ is an indeterminate form.

2. Prove the same way that $\frac{\infty}{\infty}$, $0 \cdot \infty$, and $\infty - \infty$ are also indeterminate forms.

Indeterminate?

Which of the following are indeterminate forms for limits?
If any of them isn't, then what is the value of such limit?

1. $\frac{0}{0}$
2. $\frac{0}{\infty}$
3. $\frac{0}{1}$
4. $\frac{\infty}{0}$
5. $\frac{\infty}{\infty}$
6. $\frac{1}{\infty}$
7. $0 \cdot \infty$
8. $\infty \cdot \infty$
9. $\sqrt{\infty}$
10. $\infty - \infty$
11. 1^{∞}
12. $1^{-\infty}$
13. 0^{0}
14. 0^{∞}
15. $0^{-\infty}$
16. ∞^{0}
17. ∞^{∞}
18. $\infty^{-\infty}$

Computations

Calculate:

1. $\lim_{x \to 2} \dfrac{x^2 + 2x - 6}{x^2 + 3x - 10}$
2. $\lim_{x \to 0} \dfrac{e^{2x^2} - \cos x}{x \sin x}$
3. $\lim_{x \to \infty} \dfrac{x^2}{e^x}$
4. $\lim_{x \to \infty} \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$
5. $\lim_{x \to \infty} (\sin x)\left(e^{1/x} - 1\right)$
6. $\lim_{x \to \infty} x \sin \dfrac{2}{x}$
7. $\lim_{x \to \infty} x \cos \dfrac{2}{x}$
8. $\lim_{x \to 1} \left[(\ln x) \tan \dfrac{\pi x}{2}\right]$

Careful with L'Hopital's rule

Compute

$$\lim_{x\to\infty} \frac{x+\sin x}{x}.$$

Is the following proof correct?

This is indeterminate of type $\frac{\infty}{\infty}$ so, by L'Hopital's Rule,

$$\lim_{x\to\infty} \frac{x+\sin x}{x} = \lim_{x\to\infty} \frac{1+\cos x}{1} = \lim_{x\to\infty} (1+\cos x).$$

The last limit doesn't exist so $\lim_{x\to\infty} \frac{x+\sin x}{x}$ doesn't exist. □

What is the limit?

Harder computations

Compute:

1. $\lim_{x\to\infty}\left(\frac{x+2}{x-2}\right)^{3x}$

2. $\lim_{x\to 0^+} x^x$

3. $\lim_{x\to 0}\left[\frac{\csc x}{x}-\frac{\cot x}{x}\right]$