Welcome back to MAT137!

(Section L5101, Tuesday 6-9 and Thursday 6-9)

- Please fill out the course evaluation form. You can access the evaluation form via the 'Course Evals' tab in Quercus or by clicking on the personalized link that you received automatically by e-mail through the course evaluation system
- Quiz today from 8:40 pm on Quercus.
- PS4 due on Wednesday June 17.
- **Today's topics: Concavity, Asymptotes, Curve Sketching** (Videos 6.11-6.16)
- Next day: No class! (Mixed feeling?) All the best for your midterm and the second half of the course.

Let's get started!!

Any question from previous class?

Indeterminate?

Which of the following are indeterminate forms for limits?
If any of them isn't, then what is the value of such limit?

1. $\dfrac{0}{0}$
2. $\dfrac{0}{\infty}$
3. $\dfrac{0}{1}$
4. $\dfrac{\infty}{0}$
5. $\dfrac{\infty}{\infty}$
6. $\dfrac{1}{\infty}$
7. $0 \cdot \infty$
8. $\infty \cdot \infty$
9. $\sqrt{\infty}$
10. $\infty - \infty$
11. 1^{∞}
12. $1^{-\infty}$
13. 0^{0}
14. 0^{∞}
15. $0^{-\infty}$
16. ∞^{0}
17. ∞^{∞}
18. $\infty^{-\infty}$

Computations

Calculate:

1. $$\lim_{x\to 2} \frac{x^2+2x-6}{x^2+3x-10}$$
2. $$\lim_{x\to 0} \frac{e^{2x^2}-\cos x}{x\sin x}$$
3. $$\lim_{x\to\infty} \frac{x^2}{e^x}$$
4. $$\lim_{x\to\infty} \frac{e^x+e^{-x}}{e^x-e^{-x}}$$
5. $$\lim_{x\to\infty} (\sin x)\left(e^{1/x}-1\right)$$
6. $$\lim_{x\to\infty} x\sin\frac{2}{x}$$
7. $$\lim_{x\to\infty} x\cos\frac{2}{x}$$
8. $$\lim_{x\to 1} \left[(\ln x)\tan\frac{\pi x}{2}\right]$$

Harder computations

Compute:

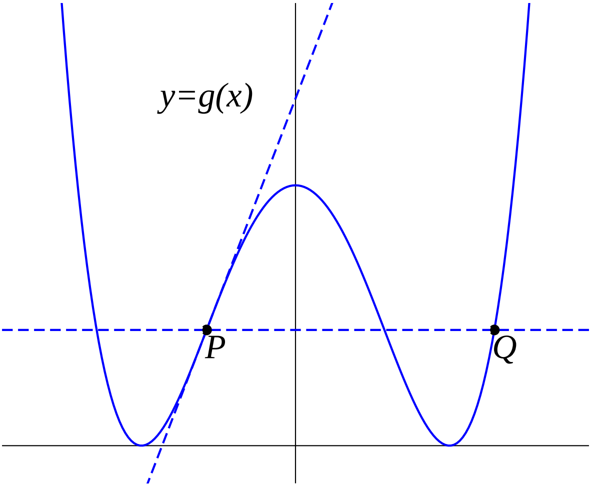
1. $$\lim_{x \to \infty} \left(\frac{x+2}{x-2} \right)^{3x}$$

2. $$\lim_{x \to 0^+} x^x$$

3. $$\lim_{x \to 0} \left[\frac{\csc x}{x} - \frac{\cot x}{x} \right]$$

Warm up: Find the coordinates of P and Q

$$g(x) = x^4 - 6x^2 + 9$$



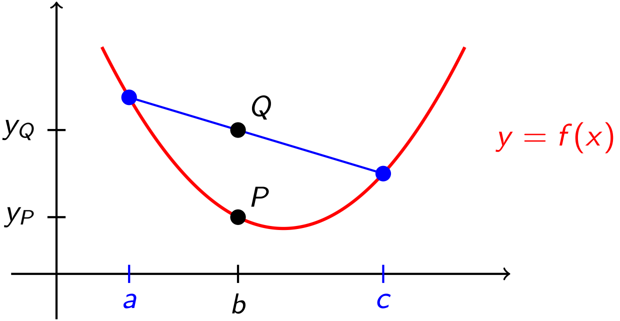
Monotonicity and concavity

Let $f(x) = xe^{-x^2/2}$.

1. Find the intervals where f is increasing or decreasing, and its local extrema.
2. Find the intervals where f is concave up or concave down, and its inflection points.
3. Calculate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
4. Using this information, sketch the graph of f.

Secants are above the graph (If time permits)

Let f be a function defined on an interval I. In Video 6.11 you learned that an alternative way to define "f is concave up on I" is to say that "the secant segments stay above the graph".



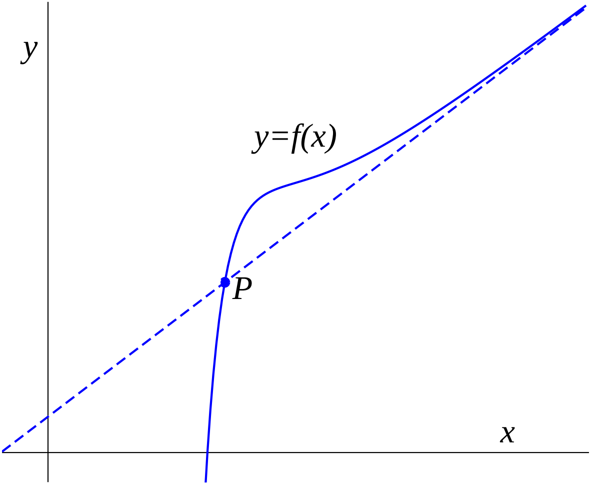
Rewrite this as a precise mathematical statement of the form

$$\text{“}\forall a, b, c \in I, \quad a < b < c \implies \boxed{\text{an inequality involving } f,\ a,\ b,\ c}\text{”}$$

Prove that f' is increasing on (a, c) implies the above inequality. (Use MVT)

Find the coordinates of P

$$f(x) = 3x + 4 + \frac{2x - 10}{x^2}$$



A function with fractional exponents

Let $h(x) = \dfrac{x^{2/3}}{(x-1)^{2/3}}$. Its first two derivatives are

$$h'(x) = \frac{-2}{3x^{1/3}(x-1)^{5/3}} \qquad h''(x) = \frac{2(6x-1)}{9x^{4/3}(x-1)^{8/3}}$$

1. Find all asymptotes of h
2. Study the monotonicity of h and local extrema
3. Study the concavity of h and inflection points
4. With this information, sketch the graph of h

Hyperbolic tangent

The function tanh, defined by

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}},$$

is called the "hyperbolic tangent".

1. Find its two asymptotes
2. Study its monotonicity
3. Study its concavity
4. With this information, sketch its graph.

An equation from the asymptotes

Construct a function f that satisfies all the following conditions at the same time.

- f is a rational function (this means it is a quotient of polynomials).
- The line $y = 1$ is an asymptote of the graph of f.
- The line $x = -1$ is an asymptote of the graph of f.

Unexpected asymptotes

Find the two asymptotes of the function

$$F(x) = x + \sqrt{x^2 + x}$$

Hint: The behaviour as $x \to \infty$ is very different from $x \to -\infty$.

Goodbye

All the best for your future!! Stay well, stay safe! :)