Welcome back to MAT137- Section L5101

- **Before next class:**
  - **Watch videos 2.14, 2.15**

Let's get started!!

Today's videos: 2.12, 2.13

Today's topic: Squeeze theorem and proofs

Any question from previous class?

Limits involving $\sin(1/x)$

The limit $\lim_{x\to 0} \sin(1/x)$:

1. DNE because the function values oscillate around 0
2. DNE because $1/0$ is undefined
3. DNE because no matter how close x gets to 0, there are x's near 0 for which $\sin(1/x) = 1$, and some for which $\sin(1/x) = -1$
4. all of the above
5. is 0

The limit $\lim_{x\to 0} x^2 \sin(1/x)$

1. does not exist because the function values oscillate around 0
2. does not exist because $1/0$ is undefined
3. does not exist because no matter how close x gets to 0, there are x's near 0 for which $\sin(1/x) = 1$, and some for which $\sin(1/x) = -1$
4. equals 0
5. equals 1

A new theorem about products

Theorem

Let $a \in \mathbb{R}$. Let f and g be functions with domain $\mathbb{R}$, except possibly a. Assume

- $\lim_{x \to a} f(x) = 0$, and
- g is bounded. This means that

$$\exists M > 0 \text{ s.t. } \forall x \neq a, |g(x)| \leq M.$$

THEN $\lim_{x \to a} [f(x)g(x)] = 0$

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1. Write down the formal definition of what you want to prove.

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1. Write down the formal definition of what you want to prove.
2. Write down what the structure of the formal proof.
3. Rough work.
4. Write down a complete formal proof.

A new squeeze

This is the Squeeze Theorem, as you know it:

The (classical) Squeeze Theorem

Let $a, L \in \mathbb{R}$.
Let f, g, and h be functions defined near a, except possibly at a.

IF
- For x close to a but not a, $h(x) \leq g(x) \leq f(x)$
- $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} h(x) = L$

THEN
- $\lim_{x \to a} g(x) = L$

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THEN
- $\lim_{x \to a} g(x) = L$

Come up with a new version of the theorem about limits being infinity. (The conclusion should be $\lim_{x \to a} g(x) = \infty$.)

Hint: Draw a picture for the classical Squeeze Theorem. Then draw a picture for the new theorem.

A new squeeze

The (new) Squeeze Theorem

Let $a \in \mathbb{R}$.
Let g and h be functions defined near a, except possibly at a.

IF
- For x close to a but not a, $h(x) \leq g(x)$
- $\lim_{x \to a} h(x) = \infty$

THEN
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1. Replace the first hypothesis with a more precise mathematical statement.

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1. Replace the first hypothesis with a more precise mathematical statement.
2. Write down the definition of what you want to prove.

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Let g and h be functions defined near a, except possibly at a.

IF
- For x close to a but not a, $\quad h(x) \leq g(x)$
- $\lim_{x \to a} h(x) = \infty$

THEN
- $\lim_{x \to a} g(x) = \infty$

1. Replace the first hypothesis with a more precise mathematical statement.
2. Write down the definition of what you want to prove.
3. Write down the structure of the formal proof.

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THEN
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1. Replace the first hypothesis with a more precise mathematical statement.
2. Write down the definition of what you want to prove.
3. Write down the structure of the formal proof.
4. Rough work
5. Write down a complete, formal proof.

A theorem about limits

Let f be a function with domain $\mathbb{R}$ such that

$$\lim_{x \to 0} f(x) = 3$$

Prove that

$$\lim_{x \to 0} [5f(2x)] = 15$$

directly from the definition of limit. Do not use any of the limit laws.

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1. Write down the formal definition of the statement you want to prove.

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1. Write down the formal definition of the statement you want to prove.
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1. Write down the formal definition of the statement you want to prove.
2. Write down what the structure of the formal proof should be, without filling the details.
3. Rough work.
4. Write down a complete proof.

Proof feedback

1. Is the structure of the proof correct?
   (First fix ε, then choose δ, then ...)
2. Did you say exactly what δ is?
3. Is the proof self-contained?
   (I do not need to read the rough work)
4. Are all variables defined? In the right order?
5. Do all steps follow logically from what comes before?
   Do you start from what you know and prove what you have to prove?
6. Are you proving your conclusion or assuming it?