Welcome back to MAT137- Section L5101

- Assignment 7 due today.
- Assignment 8 due on March 4.
- Test 4 opens on March 12.

- Monday: Watch videos 12.1-12.6
- Questions from previous class?

Let's get started!!
Today's video: 11.7 and 11.8 !!
Today's topic: The Big Theorem!

Calculations

1. $\lim_{n\to\infty} \frac{n! + 2e^n}{3n! + 4e^n}$

2. $\lim_{n\to\infty} \frac{2^n + (2n)^2}{2^{n+1} + n^2}$

3. $\lim_{n\to\infty} \frac{5n^5 + 5^n + 5n!}{n^n}$

True or False – The Big Theorem

Let $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ be positive sequences.

1. IF $a_n << b_n$, THEN $\forall m \in \mathbb{N},\ a_m < b_m$.

2. IF $a_n << b_n$, THEN $\exists m \in \mathbb{N}$ s.t. $a_m < b_m$.

3. IF $a_n << b_n$, THEN $\exists n_0 \in \mathbb{N}$ s.t.
   $\forall m \in \mathbb{N},\ m \geq n_0 \implies a_m < b_m$.

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4. IF $\forall m \in \mathbb{N}, a_m < b_m$, THEN $a_n << b_n$.

5. IF $\exists m \in \mathbb{N}$ s.t. $a_m < b_m$, THEN $a_n << b_n$.

6. IF $\exists n_0 \in \mathbb{N}$ s.t. $\forall m \in \mathbb{N},\ m \geq n_0 \implies a_m < b_m$, THEN $a_n << b_n$.

Refining the Big Theorem - 1

1. Construct a sequence $\{u_n\}_n$ such that

$$\begin{cases} \forall a < 0, & n^a << u_n \\ \forall a \geq 0, & u_n << n^a \end{cases}$$

2. Construct a sequence $\{v_n\}_n$ such that

$$\begin{cases} \forall a \leq 0, & n^a << v_n \\ \forall a > 0, & v_n << n^a \end{cases}$$

Refining the Big Theorem - 2

1. Construct a sequence $\{u_n\}_n$ such that

$$\begin{cases} \forall a < 2, & n^a << u_n \\ \forall a \geq 2, & u_n << n^a \end{cases}$$

2. Construct a sequence $\{v_n\}_n$ such that

$$\begin{cases} \forall a \leq 2, & n^a << v_n \\ \forall a > 2, & v_n << n^a \end{cases}$$

True or False - Review

1. If $\{a_n\}_{n=0}^{\infty}$ diverges and is increasing, then $\exists n \in \mathbb{N}$ s.t. $a_n > 100$.
2. If $\lim\limits_{n\to\infty} a_n = L$, then $\forall n \in \mathbb{N}$, $a_n < L + 1$.
3. If $\lim\limits_{n\to\infty} a_n = L$, then $\exists n \in \mathbb{N}$ s.t. $a_n < L + 1$.
4. If $\lim\limits_{n\to\infty} a_n = L$, then $\exists \varepsilon > 0$ s.t. $\forall n \in \mathbb{N}$, $a_n < L + \varepsilon$.
5. If $\{a_n\}_{n=0}^{\infty}$ is convergent and $b_n = a_n$ for *almost all* $n \in \mathbb{N}$, then $\{b_n\}_{n=0}^{\infty}$ is convergent.
6. If $a_n << b_n$, then $\exists n \in \mathbb{N}$ s.t. $a_n < b_n$.
7. If $a_n << b_n$, then $\forall \varepsilon > 0$, $\exists n \in \mathbb{N}$ s.t. $a_n < \varepsilon b_n$.
8. If $a_n << b_n$, then $\forall \varepsilon > 0$, $\exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N}$, $n \geq n_0 \implies a_n < \varepsilon b_n$,