Welcome back to MAT137- Section L5101

- Assignment 8 due on March 4.
- Test 4 opens on March 12.

- Wednesday: Watch videos 12.7, 12.8 (BCT)
- Questions from previous class?

Let's get started!!
Today's video: 12.1 and 12.6 !!
Today's topic: Improper integral!

Recall the definitions

1. **Type-1 improper integrals.** Let f be a bounded, continuous function on $[c, \infty)$. How do we define the improper integral

$$\int_c^\infty f(x)dx\,?$$

2. **Type-2 improper integrals.** Let f be a continuous function on $(a, b]$, possibly with $x = a$ as a vertical asymptote. How do we define the improper integral

$$\int_a^b f(x)dx\,?$$

Computation

Calculate, using the definition of improper integral

$$\int_1^{\infty} \frac{1}{x^2 + x} dx$$

Hint: $\displaystyle \frac{1}{x^2 + x} = \frac{(x+1) - (x)}{x(x+1)}$

The most important improper integrals

Use the definition of improper integral to determine for which values of $p \in \mathbb{R}$ each of the following improper integrals converges.

1. $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$
2. $\displaystyle\int_0^1 \frac{1}{x^p}\,dx$
3. $\displaystyle\int_0^\infty \frac{1}{x^p}\,dx$

Positive functions

- Let f be continuous on $[a, \infty)$. Let $A = \displaystyle\int_a^\infty f(x)dx$

  $$\text{Then } A \text{ may be } \begin{cases} \text{convergent (a number)} \\ \text{divergent} \begin{cases} \text{to } \infty \\ \text{to } -\infty \\ \text{``oscillating''} \end{cases} \end{cases}$$

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- Assume $\forall x \geq a, f(x) \geq 0$.

  Which of the four options are still possible?

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- Assume $\exists M \geq a,$ s.t. $\forall x \geq M, f(x) \geq 0.$

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A “simple” integral

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A "simple" integral

What is $\int_{-1}^{1} \frac{1}{x}\, dx$?

1. $\int_{-1}^{1} \frac{1}{x}\, dx \;=\; (\ln|x|)\Big|_{-1}^{1} \;=\; \ln|1| - \ln|-1| = 0$

2. $\int_{-1}^{1} \frac{1}{x}\, dx = 0$ because $f(x) = \frac{1}{x}$ is an odd function.

3. $\int_{-1}^{1} \frac{1}{x}\, dx$ is divergent.

What is wrong with this computation?

$$
\begin{aligned}
\int_{-1}^{1} \frac{1}{x}\,dx &= \lim_{\varepsilon\to 0^+} \left[\int_{-1}^{-\varepsilon} \frac{1}{x}\,dx + \int_{\varepsilon}^{1} \frac{1}{x}\,dx \right] \\
&= \lim_{\varepsilon\to 0^+} \left[\ln|x|\Big|_{-1}^{-\varepsilon} + \ln|x|\Big|_{\varepsilon}^{1} \right] \\
&= \lim_{\varepsilon\to 0^+} \left[\ln|-\varepsilon| - \ln|\varepsilon| \right] \\
&= \lim_{\varepsilon\to 0^+} [0] = 0
\end{aligned}
$$