Welcome back to MAT137!

(Section L5101, Tuesday 6-9 and Thursday 6-9)

- I am still Sourav!!
- My e-mail: ssarkar@math.toronto.edu (Please include MAT137 in the subject line)
- My Office Hours: Mondays, 5:00-7:00 (check Quercus for any last minute change)
- You will find all the relevant Zoom links on Quercus
- Have you seen the section webpage http://www.math.toronto.edu/ssarkar/1372020.html? This will tell you which videos to watch and also have the slides uploaded a few hours before class.
- Everything else (problem sets, tutorials...) you will find on Quercus. Regularly check Quercus and your emails for any update on the course.
- Have you read the course syllabus (all of it!)?

Reminder: Things to do

- Have you joined Piazza?
- Have you reviewed precalculus? http://uoft.me/precalc
- Have you registered in Gradescope?
- Have you enrolled in a tutorial?
- **Problem set 1** has been posted on Quercus. It is due on Wednesday, May 13th. (You need to submit the solutions on Gradescope)
- **For next day's lecture, watch videos 2.1 through 2.6.**
- **Today's topics: Conditionals, Definitions, Proofs, Induction.** (Videos 1.7 – 1.15)

How to work?

- Open today's slides alongside Zoom.
- Take notes and solve problems like in-person class.
- Listen and Watch. No distractions!
- **Mute your mic and camera to avoid lag**. Please, without exception!

How to participate?

- Do the activities as we go
- Reply to Polls!
- Use the chat if you have any question/doubt and when you give an answer
- Discuss in Breakout rooms. Socialize with your peers in the rooms, discuss math (use the audio/video).
- Ask questions, answer them (using the chat only!). Don't be afraid/shy to ask ("silly"!!) questions or be wrong in class. No one is here to judge you. And you shouldn't be here to judge others either. **Certainly, I will not!**

Let's get started!!

Any question from previous class?

Warm up: Which is correct?

The following two statements are identical *except* for the order of the two quantifiers:

$$(1) \qquad \forall x \in \mathbb{R}, \exists y \in \mathbb{R} \text{ such that } x = y.$$

$$(2) \qquad \exists y \in \mathbb{R} \text{ such that } \forall x \in \mathbb{R},\ x = y$$

Prove your claim!

Cards

Four cards lie on the table in front of you. You know that each card has a letter on one side and a number on the other. At the moment, you can read the symbols E, P, 3, and 8 on the sides that are up.
I tell you:

> *"If a card has a vowel on one side, then*
> *it has an odd number on the other side."*

Which cards do you need to turn over in order to verify whether I am telling the truth or not?

Cards

Four cards lie on the table in front of you. You know that each card has a letter on one side and a number on the other.

Negate the following statement:

> *"If a card has a vowel on one side, then*
> *it has an odd number on the other side."*

Negation of conditionals

Write the negation of these statements:

1. If Sheldon Cooper has a brother, then he also has a sister.
2. If a student in this class has a brother, then they also have a sister.

Fun with conditionals

True or False?

1. $x > 0 \implies x \geq 0$.
2. $x \geq 0 \implies x > 0$.
3. $x = 0 \implies x \geq 0$.
4. IF $0 > 1$, THEN no one is paying attention in class.

One-to-one functions

Let f be a function with domain D.
f is *one-to-one* means that different inputs (x) must produce different outputs ($f(x)$).
Write a formal definition of “one-to-one”.

One-to-one functions

Let f be a function with domain D.
What does each of the following mean? Does any of them mean f is one-to-one?

1. $f(x_1) \neq f(x_2)$
2. $\exists x_1, x_2 \in D$, s.t. $f(x_1) \neq f(x_2)$
3. $\forall x_1, x_2 \in D,\ f(x_1) \neq f(x_2)$
4. $\forall x_1, x_2 \in D,\ x_1 \neq x_2,\ f(x_1) \neq f(x_2)$
5. $\forall x_1, x_2 \in D,\ f(x_1) \neq f(x_2) \implies x_1 \neq x_2$
6. $\forall x_1, x_2 \in D,\ x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$
7. $\forall x_1, x_2 \in D,\ f(x_1) = f(x_2) \implies x_1 = x_2$

Prove a function is one-to-one

Definition

Let f be a function with domain D.
We say f is **one-to-one** when

- $$\forall x_1, x_2 \in D,\ f(x_1) = f(x_2) \implies x_1 = x_2.$$
- OR, equivalently, $\forall x_1, x_2 \in D,\ x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$.

Exercise

Prove that $f(x) = 3x + 2$ is one-to-one.

Write the structure of your proof using the 1st definition.

How do you begin? What do you assume? What do you conclude?

Prove a function is NOT one-to one

Definition

Let f be a function with domain D.
We say f is **one-to-one** when

- $$\forall x_1, x_2 \in D,\ f(x_1) = f(x_2) \implies x_1 = x_2.$$
- OR, equivalently, $\forall x_1, x_2 \in D,\ x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$.

If I give you a function f and ask you to prove it is NOT one-to-one, then you need to prove f satisfies the **negation** of the definition.

- Write the negation of the 1st definition.
- Write the negation of the 2nd definition.
- Are they different?

Exercise

Prove that $f(x) = x^2$ is **not** one-to-one.

Proving a theorem

Theorem

Let f be a function with domain D.
IF f is increasing on D THEN f is one-to-one on D.

1. Remind yourself of the precise definition of "increasing" and "one-to-one"
2. To prove a theorem, what will you assume? What do you want to show?
3. Look at the part you want to show. Based on the definition, what is the structure of the proof?
4. Complete the proof.

Disproving a theorem

Do this as an exercise

FALSE Theorem

Let f be a function with domain D.
IF f is one-to-one on D THEN f is increasing on D.

1. This theorem is false. What do you need to do to prove it is false?
2. Prove the theorem is false.

Standard Induction and its structure

We have a statement S_n that depends on some $n \in \mathbb{Z}^+$, the set of positive integers.
WTS: $\forall n \in \mathbb{Z}^+, S_n$ is true.
The standard induction has two steps:

1. Base case: S_1 is true.
2. Induction step: $\forall n \geq 1, S_n$ is true $\implies S_{n+1}$ is true .

Write the structure of an induction proof.

What if instead...

Let S_n be a statement that depends on a positive integer n.

In each case, which statements are guaranteed to be true?

1. We have proven:
   - S_3 is true.
   - $\forall n \geq 1,\ S_n$ is true $\implies S_{n+1}$ is true .
2. We have proven:
   - S_1 is true.
   - $\forall n \geq 2,\ S_n$ is true $\implies S_{n+1}$ is true .
3. We have proven:
   - S_1 is true.
   - $\forall n \geq 1,\ S_n$ is true $\implies S_{n+3}$ is true .
4. We have proven:
   - S_1 is true.
   - $\forall n \geq 1,\ S_{n+1}$ is true $\implies S_n$ is true .

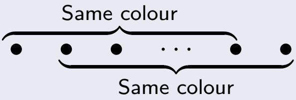
What is wrong with this proof by induction?

Theorem

$\forall N \in \mathbb{Z}$, in every set of N cars, all the cars are of the same colour.

Proof.

- **Base case.** It is clearly true for $N = 1$.
- **Induction step.**
  Assume it is true for N. I'll show it is true for $N+1$.
  Take a set of $N+1$ cars. By induction hypothesis:
  - The first N cars are of the same colour.
  - The last N cars are of the same colour.



  Hence the $N+1$ cars are all of the same colour.

$\square$