Welcome back to MAT137!

(Section L5101, Tuesday 6-9 and Thursday 6-9)

- Tutorials started this week.
- **Problem set 1** is due tomorrow. (You need to submit the solutions on Gradescope)
- **For next day's lecture, watch videos 2.7 through 2.13.**
- **Today's topics: Absolute Values, Inequalities, Limits and Their Formal Definition.** (Videos 2.1–2.6)

How to work?

- Open today's slides alongside Zoom.
- Take notes and solve problems like in-person class.
- **Mute your mic and camera to avoid lag**. Please, without exception!
- Reply to Polls!
- Use the chat if you have any question/doubt and when you give an answer
- Discuss in Breakout rooms. Socialize with your peers in the rooms, discuss math (use the audio/video).
- Ask questions, answer them (using the chat only!). Don't be afraid/shy to ask ("silly"!!) questions or be wrong in class.

Let's get started!!

Any question from previous class?

Topics: Inequalities, Absolute value and distance

Properties of inequalities

Let $a, b, c \in \mathbb{R}$. Assume $a < b$.
What can we conclude?

1. $a + c < b + c$
2. $a - c < b - c$
3. $ac < bc$
4. $a^2 < b^2$
5. $\frac{1}{a} < \frac{1}{b}$

Properties of inequalities

Let $a, b \in \mathbb{R}$. Can we always conclude

1. $|ab| = |a||b|$
2. $|a + b| = |a| + |b|$

Sets described by distance

Let $a \in \mathbb{R}$. Let $\delta > 0$.
What are the following sets? Describe them in terms of intervals.

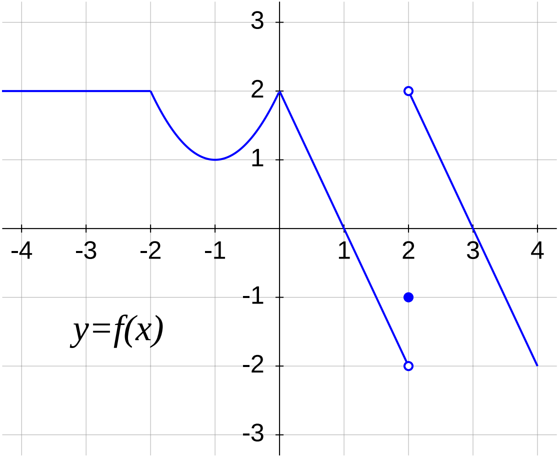
1. $A = \{x \in \mathbb{R} \,:\, |x| < \delta\}$
2. $B = \{x \in \mathbb{R} \,:\, |x| > \delta\}$
3. $C = \{x \in \mathbb{R} \,:\, |x - a| < \delta\}$
4. $D = \{x \in \mathbb{R} \,:\, 0 < |x - a| < \delta\}$

Implications

Find **all** values of A, B, and C that make the following implications true

1. $|x - 3| < 1 \implies |2x - 6| < A$
2. $|x - 3| < B \implies |2x - 6| < 1$
3. $|x - 3| < 1 \implies B < |x + 5| < C$

Limits from a graph



Find the value of

1. $\lim_{x \to 2} f(x)$
2. $\lim_{x \to 0} f(f(x))$
3. $\lim_{x \to 2} [f(x)]^2$

Floor

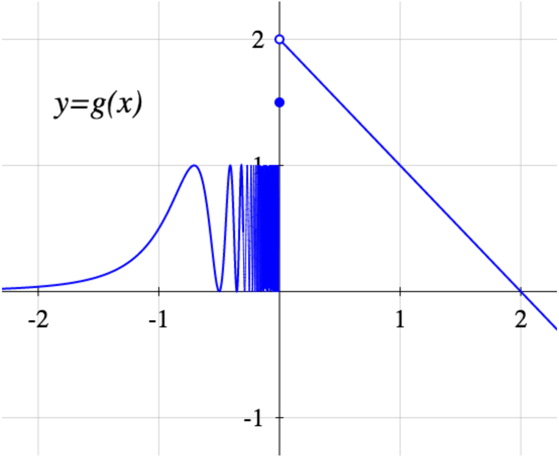
Given a real number x, we defined the **floor of** x, denoted by $\lfloor x \rfloor$, as the largest integer smaller than or equal to x. For example:

$$\lfloor \pi \rfloor = 3, \qquad \lfloor 7 \rfloor = 7, \qquad \lfloor -0.5 \rfloor = -1.$$

Sketch the graph of $y = \lfloor x \rfloor$. Then compute:

1. $\lim_{x \to 0^+} \lfloor x \rfloor$
2. $\lim_{x \to 0^-} \lfloor x \rfloor$
3. $\lim_{x \to 0} \lfloor x \rfloor$
4. $\lim_{x \to 0} \lfloor x^2 \rfloor$

More limits from a graph



Find the value of

1. $\lim_{x\to 0^+} g(x)$
2. $\lim_{x\to 0^+} \lfloor g(x) \rfloor$
3. $\lim_{x\to 0^+} g(\lfloor x \rfloor)$

4. $\lim_{x\to 0^-} g(x)$
5. $\lim_{x\to 0^-} \lfloor g(x) \rfloor$
6. $\lim_{x\to 0^-} \lfloor \frac{g(x)}{2} \rfloor$
7. $\lim_{x\to 0^-} g(\lfloor x \rfloor)$

Warm-up

Write down the formal definition of

$$\lim_{x \to a} f(x) = L.$$

Side limits

We know:

Definition

Let $L, a \in \mathbb{R}$.
Let f be a function defined at least on an interval around a, except possibly at a.

$$\lim_{x \to a} f(x) = L$$

means

$$\forall \varepsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \implies |f(x) - L| < \varepsilon.$$

Write, instead, the formal definition of

$$\lim_{x \to a^+} f(x) = L, \quad \text{and} \quad \lim_{x \to a^-} f(x) = L.$$

Infinite limits

Definition

Let $a, L \in \mathbb{R}$.

Let f be a function defined at least on an interval around a, except possibly at a.

Write formal definitions for

1. $\lim_{x \to a} f(x) = \infty$.
2. $\lim_{x \to \infty} f(x) = L$.

Hint: What does it mean mathematically to say something $\to \infty$, that is, it becomes arbitrarily large?

Existence

Write down the formal definition of the following statements:

1. $\lim_{x \to a} f(x)$ exists

2. $\lim_{x \to a} f(x)$ does not exist