Welcome back to MAT137- Section L5101

- Test 2 opens tomorrow at 3 pm.
- Assignment #5 due on Dec 20.
- **Next class: Concavity**
  - **Watch videos 6.13, 6.14**

Let's get started!!

Today's videos: 6.10-6.12

Today's topic: More indeterminate forms

Any question from previous class?

Indeterminate?

Which of the following are indeterminate forms for limits? If any of them isn't, then what is the value of such limit?

1. $\frac{0}{0}$
2. $\frac{0}{\infty}$
3. $\frac{0}{1}$
4. $\frac{\infty}{0}$
5. $\frac{\infty}{\infty}$
6. $\frac{1}{\infty}$
7. $0 \cdot \infty$
8. $\infty \cdot \infty$
9. $\sqrt{\infty}$
10. $\infty - \infty$
11. 1^{∞}
12. $1^{-\infty}$
13. 0^{0}
14. 0^{∞}
15. $0^{-\infty}$
16. ∞^{0}
17. ∞^{∞}
18. $\infty^{-\infty}$

Proving something is an indeterminate form

1. Prove that $\forall c \in \mathbb{R}$, $\exists a \in \mathbb{R}$ and functions f and g such that

$$\lim_{x \to a} f(x) = 0, \quad \lim_{x \to a} g(x) = 0, \quad \lim_{x \to a} \frac{f(x)}{g(x)} = c$$

This is how you show that $\frac{0}{0}$ is an indeterminate form.

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3. Prove that 1^{∞}, 0^0, and ∞^0 are indeterminate forms. (You will only get $c \geq 0$ this time)

Infinity minus infinity

Calculate:

1. $\lim_{x \to 0} \left[\frac{\csc x}{x} - \frac{\cot x}{x} \right]$
2. $\lim_{x \to \infty} [\ln(x+2) - \ln(3x+4)]$
3. $\lim_{x \to 1} \left[\frac{2}{x^2 - 1} - \frac{1}{x-1} \right]$
4. $\lim_{x \to -\infty} \left[\sqrt{x^2 + 3x} - \sqrt{x^2 - 3x} \right]$

More indeterminate forms

Calculate:

1. $\lim_{x \to 1} \left[(\ln x) \tan \frac{\pi x}{2} \right]$
2. $\lim_{x \to 0} \left[1 + 2 \sin(3x) \right]^{4 \cot(5x)}$
3. $\lim_{x \to \infty} \left(\frac{x+2}{x-2} \right)^{3x}$
4. $\lim_{x \to 0^+} x^x$
5. $\lim_{x \to \frac{\pi}{2}^-} (\tan x)^{\cos x}$
6. $\lim_{x \to 0} \left(\frac{\sin x}{x} \right)^{1/x^2}$

Backwards L'Hôpital

Construct a polynomial P such that

$$\lim_{x \to 1} \frac{P(x)}{e^x - e \cdot x} = \frac{1}{e}$$