Welcome back to MAT137!

(Section L5101, Tuesday 6-9 and Thursday 6-9)

- Problem set 2 due this Friday
- **For next day's lecture, watch videos 3.6, 3.7, 3.9-3.12.**
- **Today's topics: IVT, EVT, Definition of Derivative, Differentiation Rules** (Videos 2.21, 2.22, 3.1-3.5, 3.8)

How to work?

- Open today's slides alongside Zoom.
- **Mute your mic and camera to avoid lag**. Please, without exception!
- Reply to Polls!
- Use the chat if you have any question/doubt and when you give an answer
- Ask questions.

Let's get started!!

Any question from previous class?

What does it mean to say that a function f is differentiable at a?

Limit-Composition Theorem

We did not solve this in last class, but did you?

Theorem

Let $a, L \in \mathbb{R}$. IF f and g are functions such that

- A. $\lim_{x \to a} f(x) = L$
- B. g(x) is continuous at $x = L$

THEN

$$\lim_{x \to a} g(f(x)) = g(L).$$

Limits at infinity

Compute:

1. $\lim_{x\to\infty}\left(x^2-\sqrt{x^5+1}\right)$
2. $\lim_{x\to\infty}\dfrac{x^2+11}{x+1}$
3. $\lim_{x\to\infty}\dfrac{x^3+\sqrt{2x^6+1}}{2x^3+\sqrt{x^5+1}}$

Rational limits

Compute:

1. $$\lim_{x \to -3^+} \frac{x^2 - 9}{3 - 2x - x^2}$$
2. $$\lim_{x \to 1^+} \frac{x^2 - 9}{3 - 2x - x^2}$$

Existence

Prove that the equation

$$x^4 - 2x = 100$$

has at least two solutions.

Extrema

In each of the following cases, does the function f have a maximum and a minimum on the interval I?

1. $f(x) = x^2, \quad I = (-1, 1).$
2. $f(x) = \dfrac{(e^x + 2)\sin x}{x} - \cos x + 3, \quad I = [2, 6]$

Can this be proven? (Use IVT)

1. Prove that there exists a time of the day when the hour hand and the minute hand of a clock form an angle of exactly 23 degrees.

2. During a Raptors basketball game, at half time the Raptors have 51 points. Prove that at some point they had exactly 26 points.

3. Prove that at some point during Donald Trump's life, his height in centimetres was exactly equal to 10 times his weight in kilograms. Some data:
   - His height at birth: 47 cm
   - His weight at birth: 3.2 kg
   - His height today: 190 cm

True or False

Are the following two statements true?

1. IF $f(x) > 0$ for all x and $\lim_{x\to 0} f(x) = L$ exists, THEN $L > 0$.

2. IF $\lim_{x\to 0} f(x) = L$ exists, then $\lim_{x\to 0} \frac{1}{f(x)}$ exists.

Definition of a minimum

Problem. Let f be a function with domain D. Which of the following statements, if any, is a definition of

$$f \text{ has a minimum on } D.$$

If you think one of them is not a correct definition, tell me what the statements mean.

1. $\forall x \in D, \exists C \in \mathbb{R}$ such that $f(x) \geq C$.
2. $\exists C \in D$ such that $\forall x \in D$, $f(x) \geq C$.
3. $\exists C \in \mathbb{R}$ such that $\forall x \in D$, $f(x) \geq C$.
4. $\exists C \in \mathbb{R}$ such that $\forall x \in D$, $f(x) > C$.
5. $\exists c \in D$ such that $\forall x \in D$, $f(x) \geq f(c)$.
6. $\exists c \in D$ such that $\forall x \in D$, $f(x) > f(c)$.

Computing derivatives from the definition

Let f be the function defined by

$$f(x) = \frac{2}{\sqrt{x}}.$$

Compute $f'(9)$ directly from the definition of the derivative as a limit.

Derivatives.

Problem. Let f be the function defined by $f(x) = x|x|$.

Is f differentiable at 0? If so, what is its derivative?

Hint: Write f as a piecewise function.

Another way to write this function is

$$f(x) = \begin{cases} -x^2 & x < 0 \\ x^2 & x \geq 0 \end{cases}$$

Computing derivatives(Homework?)

Problem 1. Compute the derivatives of the following functions:

1. $f(x) = x^{100} + 3x^{30} - 2x^{15}$
2. $f(x) = \sqrt[3]{x} + 6$
3. $f(x) = \dfrac{4}{x^4}$
4. $f(x) = \sqrt{x}\,(1 + 2x)$
5. $f(x) = \dfrac{x^6 + 1}{x^3}$
6. $f(x) = \dfrac{x^2 - 2}{x^2 + 2}$

Problem 2. Let $0 \neq c \in \mathbb{R}$, and let f be a function that is differentiable at c. Define a new function g by:

$$g(x) = \frac{f(x)}{x^7}.$$

Compute $g'(c)$.

Prove these statements are all false with counterexamples

Let C be a curve. Let P be a point in C.

1

The line tangent to C at P intersects C at only one point: P.

2

If a line intersects C only at P,
then that line must be the tangent line to C at P.

3

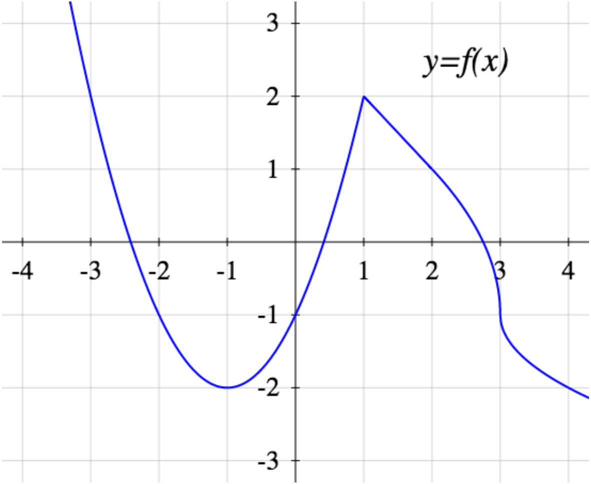
The tangent line to C at P intersects C at P and "does not cross" C at P. (This means that, near P, it stays on one side of C.)

4

If a line intersects C at P and "does not cross" C at P,
then it is the tangent line to C at P.

Derivative from a graph

Below is the graph of the function f.
Sketch the graph of its derivative f'.



Estimations

Let f be a continuous function with domain $\mathbb{R}$.

1. We know $f(4) = 3$ and $f(4.2) = 2.2$.
   Based only on this, give your best estimate for $f(4.1)$.

2. We know $f(4) = 3$ and $f'(4) = .5$.
   Based only on this, give your best estimate for $f(4.1)$.

3. We know $f(4) = 3$ and $f(4.1) = 4$.
   Based only on this, give your best estimate for $f'(4)$.

More estimations

Without using a calculator, estimate $\sqrt[20]{1.01}$ as well as you can.

Hint: Consider the values you know for $f(x) = \sqrt[20]{x}$ and its derivative.

True or False

Let $a \in \mathbb{R}$.
Let f be a function with domain $\mathbb{R}$.
Assume f is differentiable at a.
What can we conclude?

1. $f(a)$ is defined.
2. $\lim\limits_{x \to a} f(x)$ exists.
3. f is continuous at a.
4. $f'(a)$ exists.
5. $\lim\limits_{x \to a} f'(x)$ exists.
6. f' is continuous at a.

We will see a counterexample for (5) and (6) in next class!