Welcome back to MAT137!

(Section L5101, Tuesday 6-9 and Thursday 6-9)

- **For next day's lecture, watch videos 3.13-3.20, 4.1, 4.2.**
- **Today's topics: Proof of Differentiation Rules, Chain Rule, Trig. Derivatives, Implicit Differentiation** (Videos 3.6, 3.7, 3.9-3.12)

How to work?

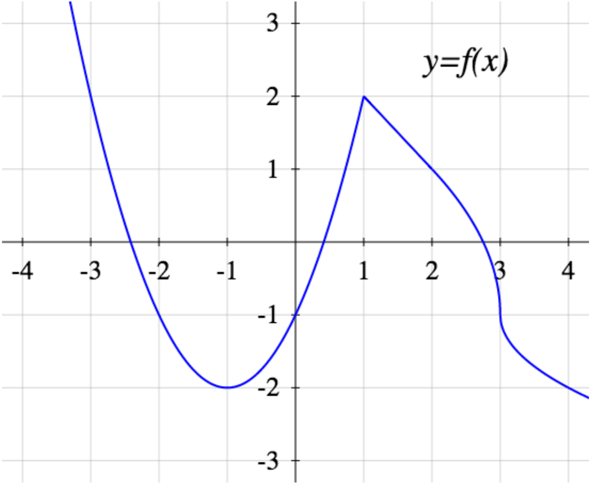
- Open today's slides alongside Zoom.
- **Mute your mic and camera to avoid lag**. Please, without exception!
- Reply to Polls!
- Use the chat if you have any question/doubt and when you give an answer
- Ask questions.

Let's get started!!

Any question from previous class?

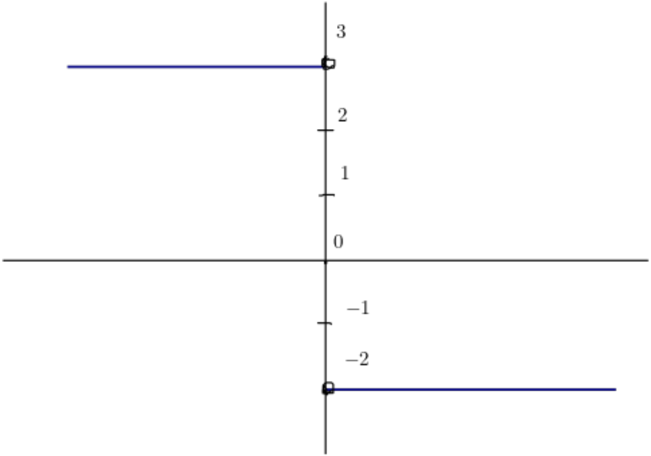
Derivative from a graph

Below is the graph of the function f.
Sketch the graph of its derivative f'.



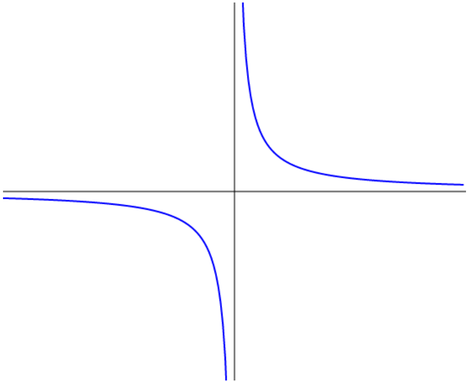
From the derivative to the function

1. Sketch the graph of a continuous function with $f(0) = 0$ whose derivative has the graph below



From the derivative to the function-2

1. Sketch the graph of a continuous function whose derivative has the graph below
2. Sketch the graph of a non-continuous function whose derivative has the graph below



Estimations

Let f be a continuous function with domain $\mathbb{R}$.

1. We know $f(4) = 3$ and $f(4.2) = 2.2$.
   Based only on this, give your best estimate for $f(4.1)$.

2. We know $f(4) = 3$ and $f'(4) = .5$.
   Based only on this, give your best estimate for $f(4.1)$.

3. We know $f(4) = 3$ and $f(4.1) = 4$.
   Based only on this, give your best estimate for $f'(4)$.

More estimations

Without using a calculator, estimate $\sqrt[20]{1.01}$ as well as you can.

Hint: Consider the values you know for $f(x) = \sqrt[20]{x}$ and its derivative.

Two simple theorems

Let $a \in \mathbb{R}$ and f be a function defined in a neighborhood of a. Prove the following.

1. IF f is differentiable at a, THEN f is continuous at a.

2. IF f is continuous at a and $f(a) \neq 0$, THEN there exists an open interval around a such that $f(x) \neq 0$ for all x in that open interval.

Proving the quotient rule.

Recall the quotient rule for derivatives from the videos, which I'll state formally here:

Theorem

Let $c \in \mathbb{R}$. Let f and g be functions defined at c and near c, and assume that $g(x) \neq 0$ for all x near c.

Define a function h by $h(x) = \dfrac{f(x)}{g(x)}$.

If f and g are differentiable at c, then h is differentiable at c, and

$$h'(c) = \frac{f'(c)g(c) - f(c)g'(c)}{[g(c)]^2}.$$

First, use the definition of $h'(c)$ to write down the limit you need to prove.

Then prove it.

Recall a trick from the product rule.

In order to prove the product rule, we had to compute a similar limit, and to do that we did a simple "trick" of adding zero in a creative way:

$$\begin{aligned}
&\frac{f(x)g(x) - f(c)g(c)}{x - c} \\
=&\frac{f(x)g(x) - f(c)g(x) + f(c)g(x) - f(c)g(c)}{x - c} \\
=&\frac{f(x) - f(c)}{x - c}\, g(x) + f(c)\frac{g(x) - g(c)}{x - c}
\end{aligned}$$

A similar (but not identical) trick will help you with this proof.

Be careful to explicitly justify any limits you evaluate in your proof.

Check your proof of the quotient rule

1. Did you use the *definition* of the derivative?
2. Are there only equations and no words? If so, you haven't written a proof.
3. Does every step follow logically from the previous steps (with explanation)?
4. Did you assume anything you couldn't assume?
5. Did you assume at any point that a function is differentiatiable? If so, did you justify it?
6. Did you assume at any point that a function is continuous? If so, did you justify it?

If you answered "no" to Q6 above, your proof cannot be fully correct.

Computations

Compute the derivative of

1. $f(x) = (2x^2 + x + 1)^8$
2. $f(x) = \dfrac{1}{\left(x + \sqrt{x^2 + x}\right)^{2019}}$

Derivatives of $(f \circ g)$

Assume f and g are functions that have all their derivatives.

Find formulas for

1. $(f \circ g)'(x)$
2. $(f \circ g)''(x)$

in terms of the values of f, g and their derivatives.

Hint: The first one is simply the chain rule.

Derivative of cos

Let

$$g(x) = \cos x.$$

Obtain and prove a formula for its derivative directly from the definition of derivative as a limit.

Hint: Imitate the derivation in Video 3.11.
If you need a trig identity that you do not know, google it or ask your neighbor.

Derivatives of the other trig functions

Using all the basic differentiation rules, as well as

$$\frac{d}{dx}\sin x = \cos x, \qquad \frac{d}{dx}\cos x = -\sin x,$$

quickly obtain and prove formulas for the derivatives of tan, cot, sec, and csc.

More computations!

Compute the derivatives of:

1. $f(x) = \tan(3x^2 + 1)$
2. $f(x) = (\cos x)(\sin 2x)(\tan 3x)$
3. $f(x) = \cos(\sin(\tan x))$
4. $f(x) = \cos\left(3x + \sqrt{1 + \sin^2 x^2}\right)$

Implicit differentiation

The equation

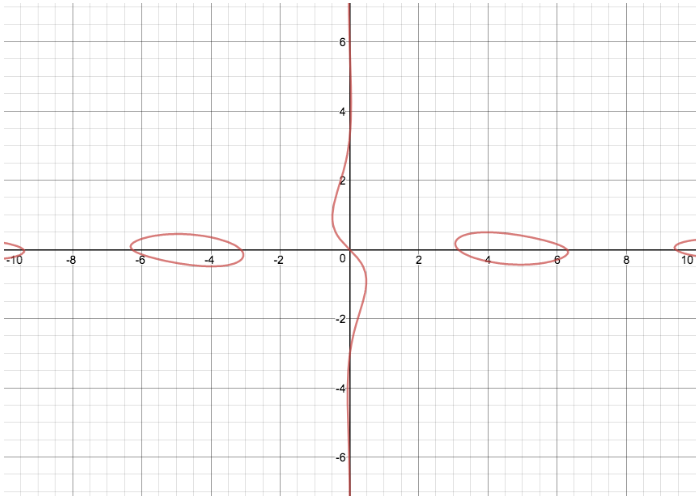
$$\sin(x + y) + xy^2 = 0$$

▸ graph

defines a function $y = h(x)$ near $(0, 0)$.

Using implicit differentiation, compute

1. $h(0)$
2. $h'(0)$
3. $h''(0)$


6
4
2
0
-2
-4
-6
-10
-8
-6
-4
-2
2
4
6
8
10

A pesky function

Let $h(x) = x^2 \sin \dfrac{1}{x}$.

1. Calculate $h'(x)$ for any $x \neq 0$.
2. Using the definition of derivative, calculate $h'(0)$.
3. Is h continuous at 0?
4. Is h differentiable at 0?
5. Is h' continuous at 0?

Hint: The last two questions have different answers.