Welcome back to MAT137!

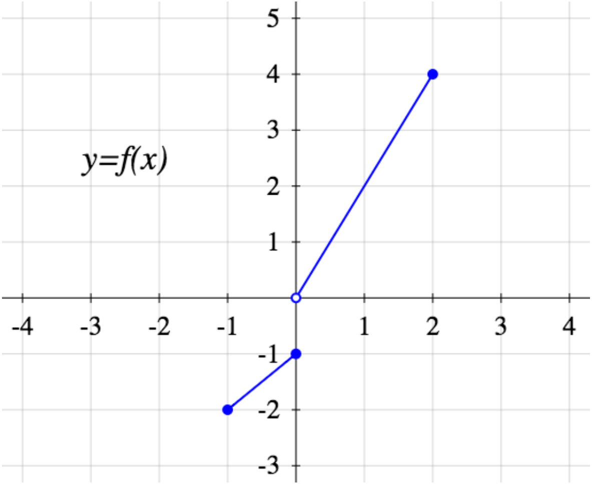
(Section L5101, Tuesday 6-9 and Thursday 6-9)

- Quiz 2 Version 2 at the end of today's class (from 9-9:20 pm) (not necessary at all)!
- PS3 is due on Friday.
- **For next day's lecture, watch videos 5.5 - 5.12.**
- **Today's topics: Inverse Functions, One-to-One Functions, Inverse Trig. Functions, Local Extrema** (Videos 4.3-4.8, 5.2-5.4)

Let's get started!!

Any question from previous class?

Inverse function from a graph



Calculate:

1. $f(2)$
2. $f(0)$
3. $f^{-1}(2)$
4. $f^{-1}(0)$
5. $f^{-1}(-1)$

Absolute value and inverses

Let

$$h(x) = x|x| + 1$$

1. Calculate $h^{-1}(-8)$.
2. Find an equation for $h^{-1}(x)$.
3. Sketch the graphs of h and h^{-1}.
4. Verify that for every $x \in \boxed{???}$, $h(h^{-1}(x)) = x$
5. Verify that for every $x \in \boxed{???}$, $h^{-1}(h(x)) = x$.

Composition and inverses

Assume all functions here have domain $\mathbb{R}$.

Let f and g be functions. Assume they each have an inverse.

Is $(f \circ g)^{-1} = f^{-1} \circ g^{-1}$?

- If YES, prove it.
- If NO, fix the statement.

If you're stuck, experiment with the functions

$$f(x) = x + 1, \qquad g(x) = 2x.$$

Composition of one-to-one functions #1

Assume all functions here have domain $\mathbb{R}$. Prove:

Theorem A

Let f and g be functions.
IF f and g are one-to-one THEN $f \circ g$ is one-to-one.

Suggestion:

1. Write the definition of what you want to prove.
2. Figure out the formal structure of the proof.
3. Complete the proof (use the hypotheses!)

Composition of one-to-one functions #2

Assume all functions here have domain $\mathbb{R}$.

Prove the following theorem.

Theorem B

Let f and g be functions.
IF $f \circ g$ is one-to-one THEN g is one-to-one.

Suggestion:

1. Rewrite the "$P \Rightarrow Q$" statement as an equivalent "not $Q \Rightarrow$ not P" statement. You will prove that.
2. Write the definition of the hypotheses and conclusion.
3. Write the proof.
4. You can also prove this directly.

Derivatives of the inverse function (Homework)

Let f be a one-to-one function.
Let $a, b \in \mathbb{R}$ be such that $b = f(a)$.

1. Review

Obtain a formula for $(f^{-1})'(b)$ in terms of $f'(a)$

Hint: This was done in the videos.
Take $\dfrac{d}{dy}$ of both sides $f(f^{-1}(y)) = y$

2. Careful

Obtain a formula for $(f^{-1})''(b)$ in terms of $f'(a)$ and $f''(a)$

3. Challenge

Obtain a formula for $(f^{-1})'''(b)$ in terms of $f'(a)$, $f''(a)$, and $f'''(a)$

Composition of one-to-one functions #3 (Homework)

Assume all functions here have domain $\mathbb{R}$.

Prove the following claim is FALSE with a counterexample.

Claim

Let f and g be functions.
IF $f \circ g$ is one-to-one THEN f is one-to-one.

Definition of arctan

1. Sketch the graph of tan.
2. Prove that tan is not one-to-one.
3. Select the largest interval containing 0 such that the restriction of tan to it is one-to-one. We define arctan as the inverse of this restriction. Let $x, y \in \mathbb{R}$.

$$\arctan y = x \qquad \iff \qquad ???$$

4. What is the domain of arctan? What is the range of arctan? Sketch the graph of arctan.
5. Compute

    1. $\arctan(\tan(1))$
    2. $\arctan(\tan(3))$
    3. $\arctan\left(\tan\left(\frac{\pi}{2}\right)\right)$
    4. $\arctan(\tan(-6))$
    5. $\tan(\arctan(0))$
    6. $\tan(\arctan(10))$

Derivative of arctan

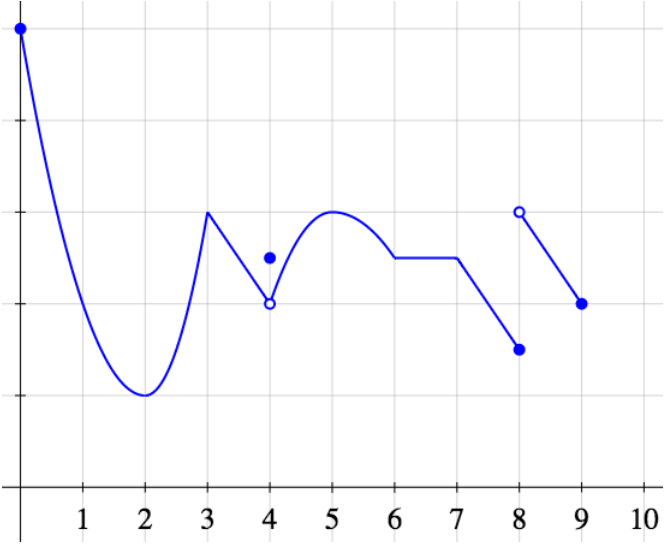
Obtain (and prove) a formula for the derivative of arctan.

Hint: Differentiate the identity

$$\forall t \in \dots \qquad \tan(\arctan(t)) = t$$

Definition of local extremum

Find the local and global extrema of the function



Where is the maximum?

We know the following about the function h:

- The domain of h is $(-4, 4)$.
- h is continuous on its domain.
- h is differentiable on its domain, except at 0.
- $h'(x) = 0 \iff x = -1$ or 1.

What can you conclude about the maximum of h?

1. h has a maximum at $x = -1$, or 1.
2. h has a maximum at $x = -1, 0$, or 1.
3. h has a maximum at $x = -4, -1, 0, 1$, or 4.
4. None of the above.

What can you conclude?

We know the following about the function f.

- f has domain $\mathbb{R}$.
- f is continuous
- $f(0) = 0$
- For every $x \in \mathbb{R}$, $f(x) \geq x$.
- $f'(0)$ exists

What can you conclude about $f'(0)$? Prove it.

Hint: Sketch the graph of f. Looking at the graph, make a conjecture.

Fractional exponents

Let $g(x) = x^{2/3}(x-1)^3$.

Find local and global extrema of g on $[-1, 2]$.