Welcome back to MAT137- Section L5101

- Assignment #1 due tomorrow.
- Mute your mic and camera to avoid lag

- **Before next class:**
  - **Watch videos 2.7, 2.8, 2.9**

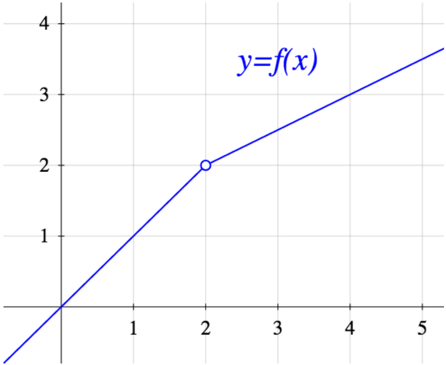
Let's get started!!

Today's videos: 2.5, 2.6

Today's topic: The formal definition of limit

Any question from previous class?

δ from a graph



1. Find one value of $\delta > 0$ s.t. $0 < |x - 2| < \delta \implies |f(x) - 2| < 0.5$
2. Find *all* values of $\delta > 0$ s.t. $0 < |x - 2| < \delta \implies |f(x) - 2| < 0.5$

Warm-up

Write down the formal definition of

$$\lim_{x \to a} f(x) = L.$$

Side limits

Recall

Let $L, a \in \mathbb{R}$.
Let f be a function defined at least on an interval around a, except possibly at a.

$$\lim_{x \to a} f(x) = L$$

means

$$\forall \varepsilon > 0, \exists \delta > 0 \text{ s.t. } \quad 0 < |x - a| < \delta \implies |f(x) - L| < \varepsilon.$$

Write, instead, the formal definition of

$$\lim_{x \to a^+} f(x) = L, \quad \text{and} \quad \lim_{x \to a^-} f(x) = L.$$

Infinite limits

Definition

Let $a \in \mathbb{R}$.
Let f be a function defined at least on an interval around a, except possibly at a.
Write a formal definition for

$$\lim_{x \to a} f(x) = \infty.$$

Infinite limits - 2

Which one(s) is the definition of $\lim_{x \to a} f(x) = \infty$?

1. $\forall M \in \mathbb{R}, \exists \delta > 0$ s.t. $0 < |x - a| < \delta \implies f(x) > M$
2. $\forall M \in \mathbb{Z}, \exists \delta > 0$ s.t. $0 < |x - a| < \delta \implies f(x) > M$
3. $\forall M > 0, \exists \delta > 0$ s.t. $0 < |x - a| < \delta \implies f(x) > M$
4. $\forall M > 5, \exists \delta > 0$ s.t. $0 < |x - a| < \delta \implies f(x) > M$
5. $\forall M \in \mathbb{R}, \exists \delta > 0$ s.t. $0 < |x - a| < \delta \implies f(x) \geq M$

Related implications

Let $a \in \mathbb{R}$. Let f be a function. Assume we know

$$0 < |x - a| < 0.1 \implies f(x) > 100$$

1. Which values of $M \in \mathbb{R}$ satisfy ... ?

$$0 < |x - a| < 0.1 \implies f(x) > M$$

Related implications

Let $a \in \mathbb{R}$. Let f be a function. Assume we know

$$0 < |x - a| < 0.1 \quad \Longrightarrow \quad f(x) > 100$$

1. Which values of $M \in \mathbb{R}$ satisfy ... ?

$$0 < |x - a| < 0.1 \quad \Longrightarrow \quad f(x) > M$$

2. Which values of $\delta > 0$ satisfy ... ?

$$0 < |x - a| < \delta \quad \Longrightarrow \quad f(x) > 100$$

Strict or non-strict inequality?

Let f be a function with domain $\mathbb{R}$. One of these statements implies the other. Which one?

1. $\forall M \in \mathbb{R}, \exists N \in \mathbb{R}$ s.t. $x > N \implies f(x) > M$
2. $\forall M \in \mathbb{R}, \exists N \in \mathbb{R}$ s.t. $x > N \implies f(x) \geq M$

More negation

Let f be a function with domain $\mathbb{R}$. Write the negation of the statement:

$$\text{IF } \ 2 < x < 4, \qquad \text{THEN } \ 1 < f(x) < 3.$$

Existence

Write down the formal definition of the following statements:

1. $\lim_{x \to a} f(x) = L$
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x)$ does not exist