Welcome back to MAT137- Section L5101

- Assignment #6 due today.
- Test 3 on February 5.
- Watch videos 9.4, 9.5 and 9.6 for Monday, February 1.
- Questions from previous class?

Let's get started!!

Today's videos: 9.1, 9.2 and 9.3!!

Today's topic: Substitution or chain rule backwards!.

Warm up

Calculate

$$\int \frac{\sin \sqrt{x}}{\sqrt{x}}\, dx$$

Hint: Use the substitution $u = \sqrt{x}$.

Computation practice: integration by substitution

Use substitutions to compute:

1. $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$
2. $\displaystyle\int e^x \cos\left(e^x\right) dx$
3. $\displaystyle\int \cot x\, dx$
4. $\displaystyle\int x^2\sqrt{x+1}\, dx$
5. $\displaystyle\int \frac{e^{2x}}{\sqrt{e^x+1}}\, dx$
6. $\displaystyle\int \frac{(\ln\ln x)^2}{x\ln x}\, dx$
7. $\displaystyle\int xe^{-x^2}\, dx$
8. $\displaystyle\int e^{-x^2}\, dx$

Definite integral via substitution

This final answer is right, but the write-up is WRONG. Why?

Calculate $I = \int_0^2 \sqrt{x^3+1}\, x^2 dx$

Wrong answer

Substitution: $u = x^3 + 1,\ du = 3x^2 dx.$

$$
\begin{aligned}
I &= \frac{1}{3}\int_0^2 \sqrt{x^3+1}\,(3x^2 dx) & &= \frac{1}{3}\int_0^2 u^{1/2}\, du \\
&= \frac{1}{3}\,\frac{2}{3}\, u^{3/2}\Big|_0^2 & &= \frac{2}{9}\left(x^3+1\right)^{2/3}\Big|_0^2 \\
&= \frac{2}{9}\left(2^3+1\right)^{3/2} - \frac{2}{9}\left(0+1\right)^{3/2} & &= \frac{52}{9}
\end{aligned}
$$

Integral of products of sin and cos

We want to compute

$$I = \int \sin^3 x \cos^2 x \, dx$$

1. Attempt the substitution $u = \sin x$
2. Attempt the substitution $u = \cos x$
3. One worked better than the other. Which one? Why? Finish the problem.

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3. One worked better than the other. Which one? Why? Finish the problem.
4. Assume we want to compute

$$\int \sin^n x \cos^m x \, dx$$

When will the substitution $u = \sin x$ be helpful?
When will the substitution $u = \cos x$ be helpful?

Odd functions

Theorem

Let f be a continuous function. Let $a > 0$. IF f is odd, THEN

$$\int_{-a}^{a} f(x)dx = 0$$

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Let f be a continuous function. Let $a > 0$. IF f is odd, THEN

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1. Write down the definition of "odd function".
2. Draw a picture to interpret the theorem geometrically.
3. Prove the theorem!
   Hint: Write the integral as sum of two pieces. Use a substitution to show that one of the two pieces equals minus the other.