MAT137

(Section L0501, September 30, 2019)

- **For next day's lecture, watch videos 2.12, 2.13, 2.14 and 2.15**
- Today's lecture will **assume** you have watched videos 2.10 and 2.11.

Contents

Infinite limits

Which statement(s) below are the definition of

$$\lim_{x \to a} f(x) = \infty$$

1. $\forall M \in \mathbb{R}, \exists \delta > 0$ s.t. $0 < |x - a| < \delta \implies f(x) > M$
2. $\forall M > 0, \exists \delta > 0$ s.t. $0 < |x - a| < \delta \implies f(x) > M$
3. $\forall M > 5, \exists \delta > 0$ s.t. $0 < |x - a| < \delta \implies f(x) > M$
4. $\forall M \in \mathbb{N}, \exists \delta > 0$ s.t. $0 < |x - a| < \delta \implies f(x) > M$
5. $\forall M \in \mathbb{R}, \exists \delta > 0$ s.t. $0 < |x - a| < \delta \implies f(x) \geq M$

Side limits

We know:

Definition

Let $L, a \in \mathbb{R}$.
Let f be a function defined at least on an interval around a, except possibly at a.

$$\lim_{x \to a} f(x) = L$$

means

$$\forall \varepsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \implies |f(x) - L| < \varepsilon.$$

Write, instead, the formal definition of

$$\lim_{x \to a^+} f(x) = L, \quad \text{and} \quad \lim_{x \to a^-} f(x) = L.$$

Limits at infinity

We did not do this in class, but you should try this at home

Write down the formal definitions of

1. $\lim_{x \to \infty} f(x) = L$

2. $\lim_{x \to \infty} f(x) = -\infty$

A harder proof

Goal

We want to prove that

$$\lim_{x \to 0} \left(x^3 + x^2\right) = 0 \tag{1}$$

directly from the definition.

1. Write down the formal definition of the above statement.
2. Write down what the structure of the formal proof should be, without filling the details.
3. **Rough work: What is δ?**
4. Write down a complete formal proof.

Choosing deltas again

We will go over (6) in the next class

Let us fix numbers $A, \epsilon > 0$.

1. Find a value of $\delta > 0$ such that $$|x| < \delta \Rightarrow |Ax^2| < \varepsilon$$
2. Find *many* values of $\delta > 0$ such that $$|x| < \delta \Rightarrow |Ax^2| < \varepsilon$$
3. Find a value of $\delta > 0$ such that $$|x| < \delta \Rightarrow |x+1| < 10$$
4. Find *many* values of $\delta > 0$ such that $$|x| < \delta \Rightarrow |x+1| < 10$$
5. Find a value of $\delta > 0$ such that $$|x| < \delta \Rightarrow \left\{ \begin{array}{c} |Ax^2| < \epsilon \\ |x+1| < 10 \end{array} \right\}$$
6. Find a value of $\delta > 0$ such that $$|x| < \delta \Rightarrow |x^2 + x^3| < \varepsilon$$

A harder proof

We will go over this in the next class

Goal

We want to prove that

$$\lim_{x \to 0} \left(x^3 + x^2\right) = 0 \tag{2}$$

directly from the definition.

1. Write down the formal definition of the statement (2).
2. Write down what the structure of the formal proof should be, without filling the details.
3. Rough work: What is δ?
4. **Write down a complete formal proof.**