# MAT137 (Section L0501, October 07, 2019)

- For next day's lecture, watch videos (2.18,) 2.19, 2.20, 2.21 and 2.22
- Today's lecture will assume you have watched videos 2.16, 2.17.

- Some more limit examples and limit at infinity
- 2 Last day's theorem on products
- Limits and composition
- A question from an old test (if time permits)

#### Goal

We want to prove that

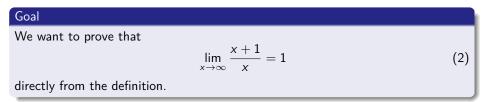
$$\lim_{x \to 1} x^3 = 1$$

directly from the definition.

- Write down the formal definition of the statement.
- Write down what the structure of the formal proof should be, without filling the details.
- **3** Rough work: What is  $\delta$ ?
- Write down a complete formal proof (Exc.).

(1)

### How to write limits at infinity



- Write down the formal definition of the statement.
- Write down what the structure of the formal proof should be, without filling the details.
- **3** Rough work. What is M?
- Write down a complete formal proof (Exc.).

Is this claim true?

### Claim

Let  $a \in \mathbb{R}$ .

Let f and g be functions defined near a.

• IF 
$$\lim_{x \to a} f(x) = 0$$

• THEN 
$$\lim_{x \to a} [f(x)g(x)] = 0$$

#### Theorem

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Let a \in \mathbb{R}.
Let f and g be functions with domain \mathbb{R}, except possibly a.
Assume
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- $\lim_{x\to a} f(x) = 0$ , and
- g is **bounded**. This means that

$$\exists M > 0 \text{ s.t. } \forall x \neq a, |g(x)| \leq M.$$

THEN  $\lim_{x\to a} [f(x)g(x)] = 0$ 

- Write down the formal definition of what you want to prove.
- 2 Write down the structure of the formal proof.
- 8 Rough work.
- Write down a complete formal proof.

#### Claim

Let  $a, L \in \mathbb{R}$ . IF f and g are functions such that

$$\lim_{x \to a} f(x) = L$$

$$\lim_{y\to L}g(y)=M$$

#### THEN

$$\lim_{x \to a} g(f(x)) = \lim_{y \to L} g(y) = M$$

#### This is FALSE.

## Two ways to fix it (Do this as an exercise)

#### FALSE claim

Let  $a, L \in \mathbb{R}$ . IF f and g are functions such that

- $\lim_{x\to a} f(x) = L$
- $\lim_{y\to L}g(y)=M$

#### THEN

$$\lim_{x\to a} g(f(x)) = \lim_{y\to L} g(y) = M$$

Recall 
$$\lim_{x \to a} f(x) = L$$
 means  

$$\begin{cases} x \text{ close to } a \quad \text{and} \quad x \neq a \end{cases} \implies f(x) \text{ close to } L$$
And  $\lim_{y \to L} g(y) = M$  means, replacing y by  $f(x)$ ,  

$$\begin{cases} f(x) \text{ close to } L \quad \text{and} \quad f(x) \neq L \end{cases} \implies g(f(x)) \text{ close to } M$$

Replace (B) with a condition to fix the theorem.

Alternatively, can you replace (A) with a condition to fix the theorem?

Sourav Sarkar

Construct a pair of functions f and g such that

- $\lim_{x\to 0} f(x) = 1$
- $\lim_{x\to 1} g(x) = 2$
- $\lim_{x\to 0} g(f(x)) = 42$

The only thing we know about the function g is that

$$\lim_{x\to 0}\frac{g(x)}{x^2}=2.$$

Use it to determine the following limits:

$$\lim_{x \to 0} \frac{g(x)}{x}$$

$$\lim_{x \to 0} \frac{g(x)}{x^4}$$

$$\lim_{x \to 0} \frac{g(3x)}{x^2}$$