MAT137

(Section L0501, October 09, 2019)

- **For next day's lecture, watch videos 3.1,3.2,3.3,3.4,3.5 and 3.8**
- Today's lecture will **assume** you have watched videos till 2.22.
- No class on Monday

Contents

Computations!

Using that $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$, compute the following limits:

1. $\lim_{x \to 2} \dfrac{\sin x}{x}$
2. $\lim_{x \to 0} \dfrac{\sin(5x)}{x}$
3. $\lim_{x \to 0} \dfrac{\tan^2(2x^2)}{x^4}$
4. $\lim_{x \to 0} \dfrac{\sin e^x}{e^x}$
5. $\lim_{x \to 0} \dfrac{1 - \cos x}{x}$
6. $\lim_{x \to 0} \dfrac{\tan^{10}(2x^{20})}{\sin^{200}(3x)}$

Limits at infinity

Compute:

1. $\lim_{x \to \infty} \left(x^7 - 2x^5 + 11 \right)$
2. $\lim_{x \to \infty} \left(x^2 - \sqrt{x^5 + 1} \right)$
3. $\lim_{x \to \infty} \dfrac{x^2 + 11}{x + 1}$
4. $\lim_{x \to \infty} \dfrac{x^2 + 2x + 3}{3x^2 + 4x + 5}$
5. $\lim_{x \to \infty} \dfrac{x^3 + \sqrt{2x^6 + 1}}{2x^3 + \sqrt{x^5 + 1}}$

Rational limits

Compute:

1. $\lim\limits_{x \to -3^+} \dfrac{x^2 - 9}{3 - 2x - x^2}$
2. $\lim\limits_{x \to 1^+} \dfrac{x^2 - 9}{3 - 2x - x^2}$

Existence

Prove that the equation

$$x^4 - 2x = 100$$

has at least two solutions.

Extrema

In each of the following cases, does the function f have a maximum and a minimum on the interval I?

1. $f(x) = x^2, \quad I = (-1, 1).$
2. $f(x) = \dfrac{(e^x + 2)\sin x}{x} - \cos x + 3, \quad I = [2, 6]$
3. $f(x) = \dfrac{(e^x + 2)\sin x}{x} - \cos x + 3, \quad I = [-2, 2]$

Can this be proven? (Use IVT)

1. Prove that there exists a time of the day when the hour hand and the minute hand of a clock form an angle of exactly 23 degrees.

2. During a Raptors basketball game, at half time the Raptors have 51 points. Prove that at some point they had exactly 26 points.

3. Prove that at some point during Sourav's life, his height in centimetres was exactly equal to 10 times his weight in kilograms. Some data:
   - His height at birth: 47 cm
   - His weight at birth: 3.2 kg
   - His height today: 172 cm

Composition of functions

We did not solve this in class, but solve this as an exercise

Claim

Let $a, L \in \mathbb{R}$. IF f and g are functions such that

A. $\lim_{x \to a} f(x) = L$

B. g(x) is continuous at $x = L$

THEN

$$\lim_{x \to a} g(f(x)) = g(L).$$