# MAT137

(Section L0501, February 05, 2020)

- **Fot today's lecture: slides 10.2, 11.1, 11.2**
- **For next day's lecture, watch videos 11.3, 11.4 .**
- Test 3 is tomorrow. All the best!
- Contents: Volumes, sequences.

# Warm up

- Differentiate $F(x) = \int_x^{x^2} e^{t^2} dt$
- What should be the substitution for $\int \sec x \tan^3 x dx$?
- What is a sequence?

# An equation for volumes by "cylindrical shells"

Let $0 < a < b$. Let $f$ be a continuous, positive function defined on $[a, b]$.

Let $R$ be the region in the first quadrant bounded between the graph of $f$ and the $x$-axis.

Find a formula for the volume of the solid of revolution obtained by rotation the region $R$ around the $y$-axis.

# Doughnut

Let $R$ be the region inside the curve with equation

$$(x-1)^2 + y^2 = 1.$$

Rotate $R$ around the line with equation $y = 4$. The resulting solid is called a *torus*.

1. Draw a picture and convince yourself that a torus looks like a doughnut.
2. Compute the volume of the torus as an integral using $x$ as the variable (“slicing method”)
3. Compute the volume of the torus as an integral using $y$ as the variable (“cylindrical shell method”)

# A crown?

Let $R$ be the region in the first quadrant bounded between the graphs of $y = x^5 - x - 2$, $x = 1.5$, $x = 2$, and the $x$-axis.

Compute the volume of the solid of revolution obtained by rotating $R$ around the $y$-axis.

# Warm up

Write a formula for the general term of these sequences

1. $\{a_n\}_{n=0}^{\infty} = \{1, 4, 9, 16, 25, \dots\}$
2. $\{b_n\}_{n=1}^{\infty} = \{1, -2, 4, -8, 16, -32, \dots\}$
3. $\{c_n\}_{n=1}^{\infty} = \left\{\frac{2}{1!}, \frac{3}{2!}, \frac{4}{3!}, \frac{5}{4!}, \dots\right\}$
4. $\{d_n\}_{n=1}^{\infty} = \{1, 4, 7, 10, 13, \dots\}$

# True or False?

Let $f$ be a function with domain at least $[1, \infty)$.
We define a sequence as $a_n = f(n)$.
Let $L \in \mathbb{R}$.

1. IF $\lim_{x \to \infty} f(x) = L$, THEN $\lim_{n \to \infty} a_n = L$.
2. IF $\lim_{n \to \infty} a_n = L$, THEN $\lim_{x \to \infty} f(x) = L$.
3. IF $\lim_{n \to \infty} a_n = L$, THEN $\lim_{n \to \infty} a_{n+1} = L$.

# Definition of limit of a sequence

Let $a_n$ be a sequence. Let $L \in \mathbb{R}$.
Which statements are equivalent to "$a_n \longrightarrow L$"?

1. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \varepsilon$
2. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N},\quad n > n_0 \implies |L - a_n| < \varepsilon$
3. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{R}$ s.t. $\forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \varepsilon$
4. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{R},\quad n \geq n_0 \implies |L - a_n| < \varepsilon$
5. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| \leq \varepsilon$
6. $\forall \varepsilon \in (0, 1),\ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \varepsilon$
7. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \dfrac{1}{\varepsilon}$
8. $\forall k \in \mathbb{Z}^+ > 0,\ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < k$
9. $\forall k \in \mathbb{Z}^+ > 0,\ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \dfrac{1}{k}$

# Definition of limit of a sequence (continued)

Let $a_n$ be a sequence. Let $L \in \mathbb{R}$.
Which statements are equivalent to "$a_n \longrightarrow L$"?

10. $\forall \varepsilon > 0$, the interval $(L - \varepsilon, L + \varepsilon)$ contains all the elements of the sequence, except the first few.
11. $\forall \varepsilon > 0$, the interval $(L - \varepsilon, L + \varepsilon)$ contains all the elements of the sequence, except finitely many.
12. $\forall \varepsilon > 0$, the interval $(L - \varepsilon, L + \varepsilon)$ contains all but finitely many of the terms of the sequence.
13. $\forall \varepsilon > 0$, the interval $[L - \varepsilon, L + \varepsilon]$ contains all but finitely many of the terms of the sequence.
14. Every interval that contains $L$ must contain all but finitely many of the terms of the sequence.
15. Every open interval that contains $L$ must contain all but finitely many of the terms of the sequence.