# MAT137

(Section L0501, October 16, 2019)

- **For next day's lecture, watch videos 3.6, 3.7, 3,9**
- Today's lecture will **assume** you have watched videos 3.1,3.2,3.3,3.4,3.5 and 3.8.
- Exam on Friday. All the best!

# How to write proof?

1. How do the quantifiers change?

2. How to write an induction proof?

# True or False

Are the following two statements true?

1. IF $f(x) > 0$ for all $x$ and $\lim_{x\to 0} f(x) = L$ exists, THEN $L > 0$.
2. IF $\lim_{x\to 0} f(x) = L$ exists, then $\lim_{x\to 0} \frac{1}{f(x)}$ exists.
3. This is False. What additional conditions do you need to make it true? Prove it.
4. (Do this as an exercise) State and prove a statement for

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}.$$

When is this true? Hint: Use the previous observation (in (3)) and the product rule.

# Definition of a minimum

**Problem.** Let $f$ be a function with domain $D$. Which of the following statements, if any, is a definition of

$$f \text{ has a minimum on } D.$$

If you think one of them is not a correct definition, find a counterexample and tell me what the statements mean.

1. $\forall x \in D$, $\exists C \in \mathbb{R}$ such that $f(x) \geq C$.
2. $\exists C \in D$ such that $\forall x \in D$, $f(x) \geq C$.
3. $\exists C \in \mathbb{R}$ such that $\forall x \in D$, $f(x) \geq C$.
4. $\exists C \in \mathbb{R}$ such that $\forall x \in D$, $f(x) > C$.
5. $\exists c \in D$ such that $\forall x \in D$, $f(x) \geq f(c)$.
6. $\exists c \in D$ such that $\forall x \in D$, $f(x) > f(c)$.

Let $f$ be the function defined by

$$f(x) = \frac{2}{\sqrt{x}}.$$

Compute $f'(9)$ directly from the definition of the derivative as a limit.

# Derivatives.

**Problem.** Let $f$ be the function defined by $f(x) = x|x|$.

Is $f$ differentiable at 0? If so, what is its derivative?

Hint: Write $f$ as a piecewise function.

Another way to write this function is

$$f(x) = \begin{cases} -x^2 & x < 0 \\ x^2 & x \geq 0 \end{cases}$$

# Computing derivatives( Solve this as an exercise)

**Problem 1.** Compute the derivatives of the following functions:

1. $f(x) = x^{100} + 3x^{30} - 2x^{15}$
2. $f(x) = \sqrt[3]{x} + 6$
3. $f(x) = \dfrac{4}{x^4}$
4. $f(x) = \sqrt{x}\,(1 + 2x)$
5. $f(x) = \dfrac{x^6 + 1}{x^3}$
6. $f(x) = \dfrac{x^2 - 2}{x^2 + 2}$

**Problem 2.** Let $0 \neq c \in \mathbb{R}$, and let $f$ be a function that is differentiable at $c$. Define a new function $g$ by:

$$g(x) = \frac{f(x)}{x^7}.$$

Compute $g'(c)$.

# Prove these statements are false with counterexamples

Let $C$ be a curve. Let $P$ be a point in $C$.

**1**

The line tangent to $C$ at $P$ intersects $C$ at only one point: $P$.

**2**

If a line intersects $C$ only at $P$,
then that line must be the tangent line to $C$ at $P$.

**3**

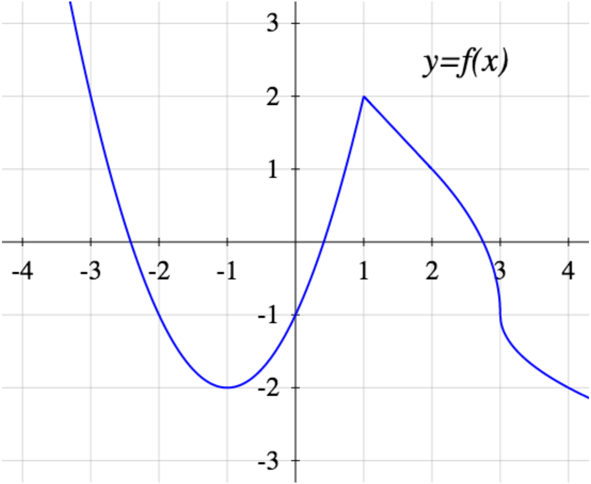
The tangent line to $C$ at $P$ intersects $C$ at $P$ and "does not cross" $C$ at $P$. (This means that, near $P$, it stays on one side of $C$.)

**4**

If a line intersects $C$ at $P$ and "does not cross" $C$ at $P$,
then it is the tangent line to $C$ at $P$.

# Derivative from a graph

Below is the graph of the function $f$.
Sketch the graph of its derivative $f\,'$.



# Estimations

Let $f$ be a continuous function with domain $\mathbb{R}$.

1. We know $f(4) = 3$ and $f(4.2) = 2.2$.
   Based only on this, give your best estimate for $f(4.1)$.

2. We know $f(4) = 3$ and $f'(4) = .5$.
   Based only on this, give your best estimate for $f(4.1)$.

3. We know $f(4) = 3$ and $f(4.1) = 4$.
   Based only on this, give your best estimate for $f'(4)$.