MAT137

(Section L0501, February 10, 2020)

- **Fot today's lecture: slides 11.3, 11.4**
- **For next day's lecture, watch videos 11.5-11.8 .**
- Contents: Properties of sequences.

True or False - Monotonic sequences vs functions

Let f be a function with domain $[0, \infty)$.
We define a sequence as $a_n = f(n)$.

1. IF f is increasing
   THEN a_n is increasing.

2. IF a_n is increasing
   THEN f is increasing.

If you think any of them is false,
prove it with a counterexample.

Monotonicty vs. boundedness vs. convergence

For each of the eight "???" boxes, construct an example sequence if possible.
If any of them is impossible, cite a theorem to justify why.

		convergent	divergent
monotonic	bounded	???	???
	unbounded	???	???
not monotonic	bounded	???	???
	unbounded	???	???

1. If a sequence is convergent, then it is bounded above.
2. If a sequence is convergent, then it is eventually monotonic.
3. If a sequence diverges and is increasing, then there exists $n \in \mathbb{N}$ such that $a_n > 100$.
4. If $\lim_{n \to \infty} a_n = L$, then $a_n < L + 1$ for all n.
5. If a sequence is non-decreasing and non-increasing, then it is convergent.
6. If a sequence isn't decreasing and isn't increasing, then it is convergent.

A sequence defined by recurrence

Consider the sequence R_n defined by

$$R_0 = 1$$
$$\forall n \in \mathbb{N}, \quad R_{n+1} = \frac{R_n + 2}{R_n + 3}$$

Compute R_1, R_2, R_3.

Is this proof correct?

Let R_n be the sequence in the previous slide.
Claim: $\{R_n\}_{n=0}^{\infty} \to -1 + \sqrt{3}$

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Proof.

- Let $L = \lim_{n\to\infty} R_n$.
- $R_{n+1} = \frac{R_n + 2}{R_n + 3}$
- $\lim_{n\to\infty} R_{n+1} = \lim_{n\to\infty} \frac{R_n + 2}{R_n + 3}$
- $L = \frac{L+2}{L+3}$
- $L(L+3) = L + 2$
- $L^2 + 2L - 2 = 0$
- $L = -1 \pm \sqrt{3}$
- L must be positive, so $L = -1 + \sqrt{3}$

□

A corrected proof

Consider the sequence R_n defined by

$$R_0 = 1$$
$$\forall n \in \mathbb{N}, \quad R_{n+1} = \frac{R_n + 2}{R_n + 3}$$

1. Prove R_n is bounded below by 0.
2. Prove R_n is decreasing (use induction)
3. Prove R_n is convergent (use a theorem)
4. Now the calculation in the previous slide is correct, and we can get the value of the limit.