MAT137

(Section L0501, October 23, 2019)

- **For next day's lecture, watch videos 3.13, 3.15, 3.16, 3.17, 3.18 (and 3.14).**
- Today's lecture will **assume** you have watched videos 3.10, 3.11, 3.12.
- Contents: 1) Chain rule 2) Trigonometric derivatives 3) Implicit differentiation 4) Applications of chain rule 5) Miscellaneous

Computations

Compute the derivative of

1. $f(x) = (2x^2 + x + 1)^8$
2. $f(x) = \dfrac{1}{\left(x + \sqrt{x^2 + x}\right)^{2019}}$

Derivatives of $(f \circ g)$

Assume f and g are functions that have all their derivatives.

Find formulas for

1. $(f \circ g)'(x)$
2. $(f \circ g)''(x)$

in terms of the values of f, g and their derivatives.

Hint: The first one is simply the chain rule.

Derivative of cos

Let

$$g(x) = \cos x.$$

Obtain and prove a formula for its derivative directly from the definition of derivative as a limit.

Hint: Imitate the derivation in Video 3.11.
If you need a trig identity that you do not know, google it or ask your neighbor.

Derivatives of the other trig functions

Using all the basic differentiation rules, as well as

$$\frac{d}{dx}\sin x = \cos x, \qquad \frac{d}{dx}\cos x = -\sin x,$$

quickly obtain and prove formulas for the derivatives of tan, cot, sec, and csc.

Computations!

Compute the derivatives of:

1. $f(x) = \tan(3x^2 + 1)$
2. $f(x) = (\cos x)(\sin 2x)(\tan 3x)$
3. $f(x) = \cos(\sin(\tan x))$
4. $f(x) = \cos\left(3x + \sqrt{1 + \sin^2 x^2}\right)$

Implicit differentiation

The equation

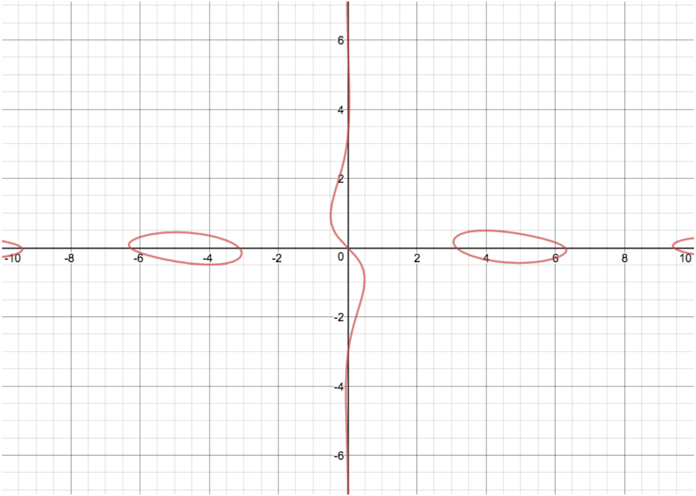
$$\sin(x + y) + xy^2 = 0$$

graph

defines a function $y = h(x)$ near $(0, 0)$.

Using implicit differentiation, compute

1. $h(0)$
2. $h'(0)$
3. $h''(0)$



6
4
2
0
-2
-4
-6
-10
-8
-6
-4
-2
2
4
6
8
10

A pesky function

Let $h(x) = x^2 \sin \frac{1}{x}$.

1. Calculate $h'(x)$ for any $x \neq 0$.
2. Using the definition of derivative, calculate $h'(0)$.
3. Is h continuous at 0?
4. Is h differentiable at 0?
5. Is h' continuous at 0?

Hint: The last two questions have different answers.

Balloon

I am inflating a spherical balloon. Below is the graph of the radius r (in cm) as a function of time t (in s). At what rate is the volume of the balloon increasing at time $4s$?