MAT137

(Section L0501, February 24, 2020)

- **For today's lecture: slides 12.1-12.6**
- **For next day's lecture, watch videos 12.7–12.10 .**
- Contents: Improper integrals.

Rapid fire: TRUE or FALSE

Each statement is about **sequences**.

1. (convergent) $\implies$ (bounded)
2. (convergent) $\implies$ (monotonic)
3. (convergent) $\implies$ (eventually monotonic)
4. (bounded) $\implies$ (convergent)
5. (monotonic) $\implies$ (convergent)
6. (bounded $+$ monotonic) $\implies$ (convergent)
7. (divergent to ∞) $\implies$ (eventually monotonic)
8. (unbounded above) $\implies$ (divergent to ∞)

Recall the definitions

1. Let f be a bounded, continuous function on $[c, \infty)$. How do we define the improper integral
$$\int_c^\infty f(x)dx\,?$$

2. Let f be a continuous function on $(a, b]$. How do we define the improper integral
$$\int_a^b f(x)dx\,?$$

Computation

Calculate, using the definition of improper integral

$$\int_1^\infty \frac{1}{x^2+x}dx$$

Hint: $\dfrac{1}{x^2+x} = \dfrac{(x+1)-x}{x(x+1)}$

The most important improper integrals

Use the definition of improper integral to determine for which values of $p \in \mathbb{R}$ each of the following improper integrals converges.

1. $\displaystyle\int_1^{\infty} \frac{1}{x^p}\, dx$
2. $\displaystyle\int_0^{1} \frac{1}{x^p}\, dx$
3. $\displaystyle\int_0^{\infty} \frac{1}{x^p}\, dx$