MAT137

(Section L0501, March 02, 2020)

- **For today's lecture: slides 13.1–13.4**
- **For next day's lecture, watch videos 13.5–13.9 .**
- Contents: Definition of series.

Rapid fire review: convergent or divergent?

1. $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$
2. $\displaystyle\int_1^\infty \frac{1}{\sqrt{x}}\,dx$
3. $\displaystyle\int_1^\infty \frac{1}{x^2+\sqrt{x}}\,dx$

Rapid fire review: convergent or divergent?

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4. $\displaystyle\int_0^1 \frac{1}{x^2}\,dx$
5. $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\,dx$
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Rapid fire review: convergent or divergent?

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6. $\displaystyle\int_0^1 \frac{1}{x^2+\sqrt{x}}\,dx$
7. $\displaystyle\int_0^\infty \frac{3}{x^2}\,dx$
8. $\displaystyle\int_0^\infty \frac{1}{\sqrt{x}}\,dx$
9. $\displaystyle\int_0^\infty \frac{1}{x^2+\sqrt{x}}\,dx$

True or false?

Are the following statements true or false for a continuous non negative function f?

- If $\lim_{x\to\infty} f(x) = L$ and $L \neq 0$, then $\int_1^\infty f(x)dx = \infty$.
- If $\lim_{x\to\infty} f(x) = 0$, then $\int_1^\infty f(x)dx < \infty$.
- If $\int_1^\infty f(x)dx < \infty$, then $\lim_{x\to\infty} f(x) = 0$.
- If $\int_1^\infty f(x)dx < \infty$ and $\lim_{x\to\infty} f(x)$ exists, then $\lim_{x\to\infty} f(x) = 0$.

Recall the definition

Define the following for the sequence $\{a_n\}_{n=1}^{\infty}$:

- $$\lim_{n\to\infty} a_n = L\,.$$
- $$\sum_{n=1}^{\infty} a_n = M\,.$$

Trig series: convergent or divergent?

1. $$\sum_{n=0}^{\infty} \sin(n\pi)$$
2. $$\sum_{n=0}^{\infty} \cos(n\pi)$$

A telescopic series

I want to calculate the value of the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + 2n}$.

1. Find a formula for the k-th partial sum

$$S_k = \sum_{n=1}^{k} \frac{1}{n^2 + 2n}.$$

Hint: Write $\frac{1}{n^2 + 2n} = \frac{A}{n} + \frac{B}{n+2}$

2. Using the definition, compute the value of

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 2n}$$

What's wrong? Fix it

Claim:

$$\sum_{n=2}^{\infty} \ln \frac{n}{n+1} = \ln 2$$

WRONG proof:

$$\begin{aligned}
\sum_{n=2}^{\infty} \ln \frac{n}{n+1} &= \sum_{n=2}^{\infty} [\ln n - \ln(n+1)] \\
&= \sum_{n=2}^{\infty} \ln(n) - \sum_{n=2}^{\infty} \ln(n+1) \\
&= (\ln 2 + \ln 3 + \ln 4 + \dots) - (\ln 3 + \ln 4 + \dots) \\
&= \ln 2
\end{aligned}$$

Help me write the problem

I want to give you a series and ask you to calculate its value from the definition. I want the sequence of partial sums $\{S_n\}_{n=1}^{\infty}$ to be

$$\forall n \geq 1,\ S_n = \frac{1}{n^2}.$$

What series should I ask you to calculate?