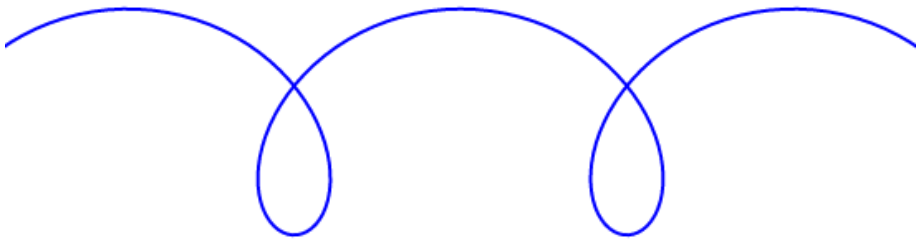
MAT137

(Section L0501, November 11, 2019)

- **For next day's lecture, watch videos 4.6, 4.7, 4.8, 5.1-5.4.**
- Today's lecture will **assume** you have watched videos 4.3, 4.4, 4.5.
- Contents: Inverse functions, one-to-one functions.

Warm up

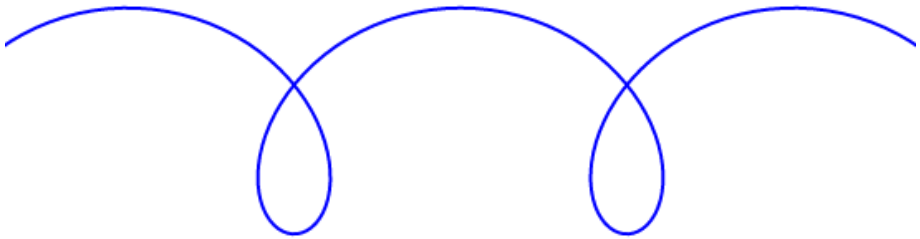
A worm is crawling across the table. The path of the worm looks something like this:



True or False?

The position of the worm in terms of time is a function.

Worm function



A worm is crawling accross the table.
For any time t, let $f(t)$ be the position of the worm.
This defines a function f.

1. What is the domain of f?
2. What is the codomain of f?
3. What is the range of f?

Composition and inverses

Assume all functions here have domain $\mathbb{R}$.

Let f and g be functions. Assume they each have an inverse.

Is $(f \circ g)^{-1} = f^{-1} \circ g^{-1}$?

- If YES, prove it.
- If NO, fix the statement.

If you're stuck, experiment with the functions

$$f(x) = x + 1, \qquad g(x) = 2x.$$

Composition of one-to-one functions #1

Assume all functions here have domain $\mathbb{R}$. Prove:

Theorem A

Let f and g be functions.
IF f and g are one-to-one THEN $f \circ g$ is one-to-one.

Suggestion:

1. Write the definition of what you want to prove.
2. Figure out the formal structure of the proof.
3. Complete the proof (use the hypotheses!)

Composition of one-to-one functions #2

Assume all functions here have domain $\mathbb{R}$.

Prove the following theorem.

Theorem B

Let f and g be functions.
IF $f \circ g$ is one-to-one THEN g is one-to-one.

Suggestion:

1. Rewrite the "$P \Rightarrow Q$" statement as an equivalent "not $Q \Rightarrow$ not P" statement. You will prove that.
2. Write the definition of the hypotheses and conclusion.
3. Write the proof.
4. You can also prove this directly.

Derivatives of the inverse function (Homework)

Let f be a one-to-one function.
Let $a, b \in \mathbb{R}$ be such that $b = f(a)$.

1. Review

Obtain a formula for $(f^{-1})'(b)$ in terms of $f'(a)$

Hint: This was done in the videos.
Take $\dfrac{d}{dy}$ of both sides $f(f^{-1}(y)) = y$

2. Careful

Obtain a formula for $(f^{-1})''(b)$ in terms of $f'(a)$ and $f''(a)$

3. Challenge

Obtain a formula for $(f^{-1})'''(b)$ in terms of $f'(a)$, $f''(a)$, and $f'''(a)$

Composition of one-to-one functions #3 (Homework)

Assume all functions here have domain $\mathbb{R}$.

Prove the following claim is FALSE with a counterexample.

Claim

Let f and g be functions.
IF $f \circ g$ is one-to-one THEN f is one-to-one.