MAT137 (Section L0501, March 04, 2020)

- For today's lecture: slides 13.5–13.9
- For next day's lecture, watch videos 13.10-13.12 .
- Contents: Properties of series.

$$\int_{1}^{\infty} \frac{1}{x^{2}} dx$$

$$\int_{1}^{\infty} \frac{1}{x} dx$$

$$\int_{1}^{\infty} \frac{1}{\sqrt{x}} dx$$

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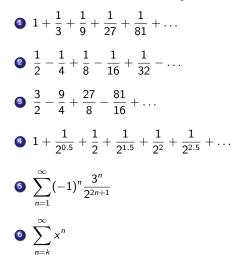
$$\int_{1}^{\infty} \frac{x+1}{x^3+2} dx$$

$$\int_{1}^{\infty} \frac{\sqrt{x^2+5}}{x^3+6} dx$$

$$\int_{1}^{\infty} \frac{x^2+3}{\sqrt{x^5+2}} dx$$

Geometric series

Calculate the value of the following series:



Proving a property about series from the definition

Let
$$\sum_{n=0}^{\infty} a_n$$
 be a series. Let $c \in \mathbb{R}$. Prove that
• IF $\sum_{n=0}^{\infty} a_n$ is convergent.
• THEN $\sum_{n=0}^{\infty} (ca_n)$ is also convergent and
 $\sum_{n=0}^{\infty} (ca_n) = c \left[\sum_{n=0}^{\infty} a_n \right].$

Write a proof directly from the definition of series.

Is $0.999999 \cdots = 1?$

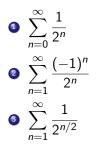
- Write the number 0.9999999... as a series *Hint*: 427 = 400 + 20 + 7.
- Occupies the first few partial sums
- Add up the series. *Hint:* it is geometric.

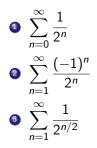
From the geometric series, we know that when |x| < 1, we can expand the function $f(x) = \frac{1}{1-x}$ as a series:

$$f(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

Find a similar way to write the following functions as series:

$$g(x) = \frac{1}{1+x^2}$$
 $h(x) = \frac{1}{2-x}$





$$\sum_{n=5}^{\infty} \frac{3^{n}}{2^{2n+1}}$$

$$\sum_{n=3}^{\infty} \frac{3^{n}}{1000 \cdot 2^{n+2}}$$

$$\sum_{n=0}^{\infty} (-1)^{n}$$

Necessary condition: TRUE or FALSE

A series $\sum_{n=0}^{\infty} a_n$ may be any one of:

- convergent
- divergent to ∞
- divergent to $-\infty$
- divergent and "oscillating"
- Give one example of each of the four results.

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- convergent
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- Give one example of each of the four results.
- Ow assume ∀n ∈ N, a_n ≥ 0. Which of the four outcomes is still possible?

Assume $\forall n \in \mathbb{N}, a_n > 0$. Consider the series $\sum_{n=0}^{\infty} a_n$. Let $\{S_n\}_{n=0}^{\infty}$ be its sequence of partial sums.

In each case, what can you conclude about the **series**? Is it convergent, divergent, or we do not know?

- **2** $\exists M \in \mathbb{R} \text{ s.t. } \forall n \in \mathbb{N}, S_n \leq M.$
- $\exists M > 0 \text{ s.t. } \forall n \in \mathbb{N}, a_n \leq M.$
- $\exists M > 0 \text{ s.t. } \forall n \in \mathbb{N}, a_n \geq M.$

Using geometric series, we showed last time that

$$g(x) = rac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}$$
 for $|x| < 1$

- **1** Compute $\frac{d}{dx} \arctan x$
- Pretend you can take antiderivatives of series just like you do with finite sums. Which series adds up to arctan x?

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- **1** Compute $\frac{d}{dx} \arctan x$
- Pretend you can take antiderivatives of series just like you do with finite sums. Which series adds up to arctan x?
- Find the value of

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)3^n}$$