MAT137

(Section L0501, November 13, 2019)

- **For next day's lecture, watch videos 5.5, 5.6.**
- Today's lecture will **assume** you have watched videos 4.6, 4.7, 4.8, 5.1-5.4.
- Contents: Inverse trigonometric functions, local extrema.

Definition of arctan

1. Sketch the graph of tan.
2. Prove that tan is not one-to-one.
3. Select the largest interval containing 0 such that the restriction of tan to it is one-to-one. We define arctan as the inverse of this restriction. Let $x, y \in \mathbb{R}$.

$$\arctan y = x \qquad \Longleftrightarrow \qquad ???$$

4. What is the domain of arctan? What is the range of arctan?
   Sketch the graph of arctan.
5. Compute

   1. $\arctan\left(\tan\left(1\right)\right)$
   2. $\arctan\left(\tan\left(3\right)\right)$
   3. $\arctan\left(\tan\left(\frac{\pi}{2}\right)\right)$
   4. $\arctan\left(\tan\left(-6\right)\right)$
   5. $\tan\left(\arctan\left(0\right)\right)$
   6. $\tan\left(\arctan\left(10\right)\right)$

Derivative of arctan

Obtain (and prove) a formula for the derivative of arctan.

Hint: Differentiate the identity

$$\forall t \in \ldots \qquad \tan(\arctan(t)) = t$$

Computation

Compute the derivative of

$$f(x) = 2x^2 \arctan(x^2) - \ln(x^4 + 1)$$

and simplify it as much as possible.

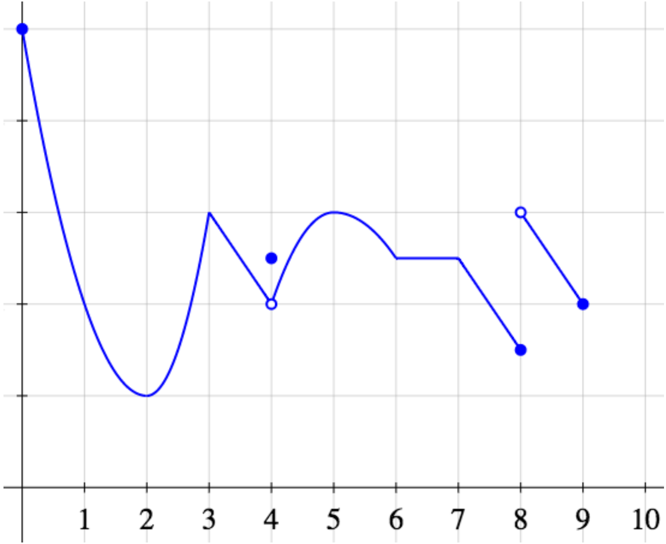
A question from an old test

Draw the graph of a function satisfying all of the following:

1. The domain of f is $\mathbb{R}$
2. f is differentiable everywhere
3. The restriction of f to $[0, \infty)$ is one-to-one and its INVERSE has a vertical tangent at 2.
4. The restriction of f to $(-\infty, 0]$ is one-to-one and its INVERSE has derivative 2 at -2.

Definition of local extremum

Find the local and global extrema of the function



Where is the maximum?

We know the following about the function h:

- The domain of h is $(-4, 4)$.
- h is continuous on its domain.
- h is differentiable on its domain, except at 0.
- $h'(x) = 0 \quad \iff \quad x = -1$ or 1.

What can you conclude about the maximum of h?

1. h has a maximum at $x = -1$, or 1.
2. h has a maximum at $x = -1, 0$, or 1.
3. h has a maximum at $x = -4, -1, 0, 1$, or 4.
4. None of the above.

What can you conclude?

We know the following about the function f.

- f has domain $\mathbb{R}$.
- f is continuous
- $f(0) = 0$
- For every $x \in \mathbb{R}$, $f(x) \geq x$.

What can you conclude about $f'(0)$? Prove it.

Answer: Either $f'(0)$ does not exist or $f'(0) = 1$.

Hint: Sketch the graph of f. Looking at the graph, make a conjecture. Imitate the proof of the Local EVT from Video 5.3.

Fractional exponents

Let $g(x) = x^{2/3}(x-1)^3$.

Find local and global extrema of g on $[-1, 2]$.