MAT137

(Section L0501, March 09, 2020)

- **For today's lecture: slides 13.10–13.12**
- **For next day's lecture, watch videos 13.13–13.17 .**
- Contents: Integral and comparison tests.

Rapid fire: For which values of $a \in \mathbb{R}$ are these series convergent?

1. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{a^n}$
2. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^a}$
3. $\displaystyle\sum_{n=0}^{\infty} a^n$
4. $\displaystyle\sum_{n=0}^{\infty} n^a$

More rapid fire: Convergent or divergent?

1. $\displaystyle\sum_{n}^{\infty} \frac{n^{10} + 17n^7 + 3}{n^{11}}$

2. $\displaystyle\sum_{n}^{\infty} \frac{\sqrt[3]{n^2 + 1} + 1}{\sqrt{n^4 + n} + n + 1}$

TRUE or FALSE

Let $\sum_{n=0}^{\infty} a_n$ be a series. Let $\{S_n\}_{n=0}^{\infty}$ be its partial-sum sequence.

1. IF the sequence $\{S_n\}_{n=0}^{\infty}$ is bounded and eventually monotonic,

   THEN the series $\sum_{n=0}^{\infty} a_n$ is convergent.

2. IF the series $\sum_{n=0}^{\infty} a_n$ converges

   THEN the sequence $\{S_n\}_{n=0}^{\infty}$ is eventually monotonic.

3. If $\forall n \in \mathbb{N}, a_n > 0$ THEN the sequence $\{S_n\}_{n=0}^{\infty}$ is increasing

4. If $\sum_{n=0}^{\infty} a_n$ is convergent THEN $\lim_{k\to\infty} \left[\sum_{n=k}^{\infty} a_n\right] = 0$

Slower questions: convergent or divergent?

1. $\displaystyle\sum_{n}^{\infty} \frac{2^n - 40}{3^n - 20}$
2. $\displaystyle\sum_{n}^{\infty} \frac{1}{n \ln n}$
3. $\displaystyle\sum_{n}^{\infty} e^{-n^2}$

Slower questions: convergent or divergent?

1. $\displaystyle\sum_n \frac{2^n - 40}{3^n - 20}$
2. $\displaystyle\sum_n \frac{1}{n \ln n}$
3. $\displaystyle\sum_n e^{-n^2}$
4. $\displaystyle\sum_n \sin^2 \frac{1}{n}$
5. $\displaystyle\sum_n \frac{1}{n(\ln n)^3}$
6. $\displaystyle\sum_n \frac{(\ln n)^{20}}{n^2}$