MAT137

(Section L0501, November 18, 2019)

- **For next day's lecture, watch videos 5.7-5.12.**
- Today's lecture will **assume** you have watched videos 5.5, 5.6.
- Contents: Rolle's Theorem.

We know the following about the function f.

- f has domain $\mathbb{R}$.
- f is continuous
- $f(0) = 0$
- For every $x \in \mathbb{R}$, $f(x) \geq x$.

What can you conclude about $f'(0)$? Prove it.

Hint: Consider the function $g(x) = f(x) - x$. How does g behave at 0?

Zeroes of functions and their derivatives

For each part, construct a function f that is differentiable on $\mathbb{R}$ and such that:

1. f has exactly 2 zeroes and f' has exactly 1 zero.
2. f has exactly 2 zeroes and f' has exactly 2 zeroes.
3. f has exactly 3 zeroes and f' has exactly 1 zero.
4. f has exactly 1 zero and f' has infinitely many zeroes.

(A sketch of a graph is good enough for each part.)

Finding the number of roots of a function

Problem. Let

$$f(x) = e^x - \sin x + x^2 + 10x.$$

How many zeroes does f have?

A nice consequence of Rolle's Theorem (Complete this; we may go over this in the next class)

Theorem

Let $a < b$ be real numbers. Let f be a function defined on $[a, b]$.

IF

- *f is continuous on $[a, b]$ and differentiable on (a, b)*
- *f is **not** injective on $[a, b]$*

THEN $\exists c \in (a, b)$ such that $f'(c) = 0$.

1. Write the definition of "f is not injective on $[a, b]$". You will need it.
2. Recall the statement of Rolle's Theorem. You will need that too.
3. Do some rough work to understand why this is true.
4. Write the proof.

Common misconceptions (Think at home)

Construct a function f satisfying all the following properties at once:

- The domain of f is $\mathbb{R}$.
- f is continuous
- $f'(0) = 0$
- f **does not** have a local extremum at 0.
- There **is no** interval centered at 0 on which f is increasing.
- There **is no** interval centered at 0 on which f is decreasing.

Hint: We have seen such a function before.