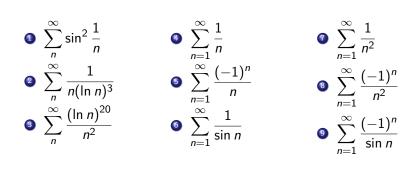
MAT137 (Section L0501, March 11, 2020)

- For today's lecture: slides 13.13–13.17
- For next day's lecture, watch videos 13.18-13.19 .
- Contents: Alternating series, conditional and absolute convergence.



True or False

Let
$$\sum_{n=0}^{\infty} a_n$$
 be a series. Let $\{S_n\}_{n=0}^{\infty}$ be its partial-sum sequence.
a IF $\lim_{n \to \infty} S_{2n}$ exists, THEN $\sum_{n=0}^{\infty} a_n$ is convergent.
b IF $\lim_{n \to \infty} S_{2n}$ exists and $\lim_{n \to \infty} a_n = 0$,
THEN $\sum_{n=0}^{\infty} a_n$ is convergent.
c IF $\sum_{n=0}^{\infty} a_n$ is convergent.
c IF $\sum_{n=0}^{\infty} a_n$ is convergent, THEN $\lim_{k \to \infty} \left[\sum_{n=k}^{\infty} a_n\right] = 0$.
c IF $\sum_{n=0}^{\infty} a_{2n}$ is convergent and $\sum_{n=0}^{\infty} a_{2n+1}$ is convergent,
THEN $\sum_{n=0}^{\infty} a_n$ is convergent.

Verify carefully the 3 hypotheses of the Alternating Series Test for

$$\sum_{n=0}^{\infty} (-1)^n \frac{n-\pi}{e^n}$$

Can we conclude it is convergent?

Estimate the sum

$$S = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$$

with an error smaller than 0.001. Write your final answer as a rational number (i.e. as a quotient of two integers).

$$\sum_{n}^{\infty} (1.1)^{n}$$

$$\sum_{n}^{n} (0.9)^{n}$$

$$\sum_{n}^{\infty} \frac{(-1)^{n}}{\ln n}$$

$$\sum_{n}^{\infty} \frac{(-1)^{n}}{e^{1/n}}$$

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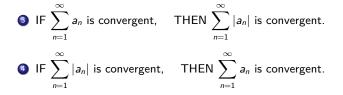
$$\sum_{n}^{\infty} \frac{1}{n^{1.1}}$$

$$\sum_{n}^{\infty} \frac{1}{n^{0.9}}$$

$$\sum_{n}^{\infty} \frac{n^3 + n^2 + 11}{n^4 + 2n - 3}$$

1 IF
$$\{a_n\}_{n=1}^{\infty}$$
 is convergent, THEN $\{|a_n|\}_{n=1}^{\infty}$ is convergent.

2 IF $\{|a_n|\}_{n=1}^{\infty}$ is convergent, THEN $\{a_n\}_{n=1}^{\infty}$ is convergent.



Positive and negative terms I

- Let $\sum a_n$ be a series.
- Call \sum (P.T.) the sum of only the positive terms.
- Call $\sum ({\sf N}.{\sf T}.)$ the sum of only the negative terms.

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	IF \sum (P.T.) is	AND \sum (N.T.) is	THEN $\sum a_n$ may be
1	CONV	CONV	
2	∞	CONV	
3	CONV	$-\infty$	
4	∞	$-\infty$	

Positive and negative terms II

- Let $\sum a_n$ be a series.
- Call \sum (P.T.) the sum of only the positive terms.
- Call $\sum ({\sf N}.{\sf T}.)$ the sum of only the negative terms.

Positive and negative terms II

- Let $\sum a_n$ be a series.
- Call \sum (P.T.) the sum of only the positive terms.
- Call \sum (N.T.) the sum of only the negative terms.

		\sum (P.T.) may be	\sum (N.T.) may be
1	In general		
2	If $\sum a_n$ is CONV		
3	If $\sum a_n$ is ABS CONV		
4	If $\sum a_n$ is COND CONV		
5	$lf\sum a_n=\infty$		
6	If $\sum a_n$ is DIV oscillating		

Construct a series of the form

$$\sum_{n=1}^{\infty} (-1)^n b_n$$

such that

•
$$b_n > 0$$
 for all $n \ge 1$

•
$$\lim_{n \to \infty} b_n = 0$$

• the series $\sum_{n=1}^{\infty} (-1)^n b_n$ is divergent.