MAT137

(Section L0501, March 11, 2020)

- **For today's lecture: slides 13.13–13.17**
- **For next day's lecture, watch videos 13.18–13.19 .**
- Contents: Alternating series, conditional and absolute convergence.

Rapid fire: convergent or divergent?

1. $\sum_{n}^{\infty} \sin^2 \frac{1}{n}$
2. $\sum_{n}^{\infty} \frac{1}{n(\ln n)^3}$
3. $\sum_{n}^{\infty} \frac{(\ln n)^{20}}{n^2}$
4. $\sum_{n=1}^{\infty} \frac{1}{n}$
5. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$
6. $\sum_{n=1}^{\infty} \frac{1}{\sin n}$
7. $\sum_{n=1}^{\infty} \frac{1}{n^2}$
8. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$
9. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sin n}$

True or False

Let $\sum_{n=0}^{\infty} a_n$ be a series. Let $\{S_n\}_{n=0}^{\infty}$ be its partial-sum sequence.

1. IF $\lim_{n\to\infty} S_{2n}$ exists, THEN $\sum_{n=0}^{\infty} a_n$ is convergent.
2. IF $\lim_{n\to\infty} S_{2n}$ exists and $\lim_{n\to\infty} a_n = 0$,

   THEN $\sum_{n=0}^{\infty} a_n$ is convergent.
3. IF $\sum_{n=0}^{\infty} a_n$ is convergent, THEN $\lim_{k\to\infty} \left[\sum_{n=k}^{\infty} a_n \right] = 0$.
4. IF $\sum_{n=0}^{\infty} a_{2n}$ is convergent and $\sum_{n=0}^{\infty} a_{2n+1}$ is convergent,

   THEN $\sum_{n=0}^{\infty} a_n$ is convergent.

An AST example

Verify carefully the 3 hypotheses of the Alternating Series Test for

$$\sum_{n=0}^{\infty}(-1)^n \frac{n-\pi}{e^n}$$

Can we conclude it is convergent?

Estimation

Estimate the sum

$$S = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$$

with an error smaller than 0.001. Write your final answer as a rational number (i.e. as a quotient of two integers).

Rapid fire: convergent or divergent?

1. $\sum_{n}^{\infty}(1.1)^n$
2. $\sum_{n}^{\infty}(0.9)^n$
3. $\sum_{n}^{\infty}\frac{(-1)^n}{\ln n}$
4. $\sum_{n}^{\infty}\frac{(-1)^n}{e^{1/n}}$

Rapid fire: convergent or divergent?

1. $\sum_{n}^{\infty}(1.1)^n$
2. $\sum_{n}^{\infty}(0.9)^n$
3. $\sum_{n}^{\infty}\frac{(-1)^n}{\ln n}$
4. $\sum_{n}^{\infty}\frac{(-1)^n}{e^{1/n}}$
5. $\sum_{n}^{\infty}\frac{1}{n^{1.1}}$
6. $\sum_{n}^{\infty}\frac{1}{n^{0.9}}$
7. $\sum_{n}^{\infty}\frac{n^3+n^2+11}{n^4+2n-3}$

TRUE or FALSE: absolute values

1. IF $\{a_n\}_{n=1}^{\infty}$ is convergent, THEN $\{\,|a_n|\,\}_{n=1}^{\infty}$ is convergent.
2. IF $\{\,|a_n|\,\}_{n=1}^{\infty}$ is convergent, THEN $\{a_n\}_{n=1}^{\infty}$ is convergent.
3. IF $\sum_{n=1}^{\infty} a_n$ is convergent, THEN $\sum_{n=1}^{\infty} |a_n|$ is convergent.
4. IF $\sum_{n=1}^{\infty} |a_n|$ is convergent, THEN $\sum_{n=1}^{\infty} a_n$ is convergent.

Positive and negative terms I

- Let $\sum a_n$ be a series.
- Call $\sum$(P.T.) the sum of only the positive terms.
- Call $\sum$(N.T.) the sum of only the negative terms.

Positive and negative terms I

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	IF $\sum$(P.T.) is...	AND $\sum$(N.T.) is...	THEN $\sum a_n$ may be...
1	CONV	CONV	
2	∞	CONV	
3	CONV	$-\infty$	
4	∞	$-\infty$	

Positive and negative terms II

- Let $\sum a_n$ be a series.
- Call $\sum$(P.T.) the sum of only the positive terms.
- Call $\sum$(N.T.) the sum of only the negative terms.

Positive and negative terms II

- Let $\sum a_n$ be a series.
- Call $\sum(\text{P.T.})$ the sum of only the positive terms.
- Call $\sum(\text{N.T.})$ the sum of only the negative terms.

		$\sum(\text{P.T.})$ may be...	$\sum(\text{N.T.})$ may be...
1	In general		
2	If $\sum a_n$ is CONV		
3	If $\sum a_n$ is ABS CONV		
4	If $\sum a_n$ is COND CONV		
5	If $\sum a_n = \infty$		
6	If $\sum a_n$ is DIV oscillating		

A counterexample to the AST?

Construct a series of the form

$$\sum_{n=1}^{\infty}(-1)^n b_n$$

such that

- $b_n > 0$ for all $n \geq 1$
- $\lim_{n\to\infty} b_n = 0$
- the series $\sum_{n=1}^{\infty}(-1)^n b_n$ is divergent.