MAT137

(Section L0501, January 6, 2020)

- **Welcome back and Happy New Year!! :)**
- **For next day's lecture, watch videos 7.3–7.9 (hard!).**
- Contents: Sums and sigmas.

Warm-up: sums

Recall:

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n$$

Compute

1. $\sum_{i=2}^{4} (2i + 1)$
2. $\sum_{i=2}^{4} 2i + 1$
3. $\sum_{j=2}^{4} (2i + 1)$

Write these sums with sigma notation

1. $1^5 + 2^5 + 3^5 + 4^5 + \ldots + 100^5$

2. $\frac{2}{4^2} + \frac{2}{5^2} + \frac{2}{6^2} + \frac{2}{7^2} + \ldots + \frac{2}{N^2}$

3. $\cos 0 - \cos 1 + \cos 2 - \cos 3 + \ldots \pm \cos(N+1)$

4. $\frac{1}{0!} + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots + \frac{1}{(2N)!}$

5. $\frac{1}{1!} - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \ldots + \frac{1}{81!}$

6. $\frac{x^2}{3!} + \frac{2x^3}{4!} + \frac{3x^4}{5!} + \frac{4x^5}{6!} + \ldots + \frac{999x^{1000}}{1001!}$

Re-writing sums

1. $$\sum_{i=1}^{100} \tan i - \sum_{i=1}^{50} \tan i = \sum_{???}^{???} ???$$

2. $$\sum_{i=1}^{N} (2i-1)^5 = \sum_{i=0}^{N-1} ???$$

3. $$\left[\sum_{k=1}^{N} x^k \right] + \left[\sum_{k=0}^{N} k\ x^{k+1} \right] = \left[\sum_{k=???}^{???} ??? x^k \right] + ???$$

Telescopic sum

Calculate the exact value of

$$\sum_{i=1}^{2{,}019} \left[\frac{1}{i} - \frac{1}{i+1} \right]$$

Hint: Write down the first few terms.

Calculate the exact value of

$$\sum_{i=1}^{10{,}000} \frac{1}{i(i+1)}$$

Double sums

Compute:

1. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{N} 1$
2. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{i} 1$
3. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{i} i$
4. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{i} k$
5. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{i} (ik)$

Useful formulas: (Google how to calculate these sums!)

$$\sum_{j=1}^{N} j = \frac{N(N+1)}{2}, \qquad \sum_{j=1}^{N} j^2 = \frac{N(N+1)(2N+1)}{6}, \qquad \sum_{j=1}^{N} j^3 = \frac{N^2(N+1)^2}{4}$$

Fubini-Tonelli (Do this as homework)

- $A_{i,k}$ is a function of 2 variables.
  For example, $A_{i,k} = \dfrac{i}{k + i^2}$.

- Decide what to write instead of each "?" so that the following identity is true:

$$\sum_{i=1}^{N} \sum_{k=1}^{i} A_{i,k} = \sum_{k=?}^{?} \sum_{i=?}^{?} A_{i,k}$$