MAT137

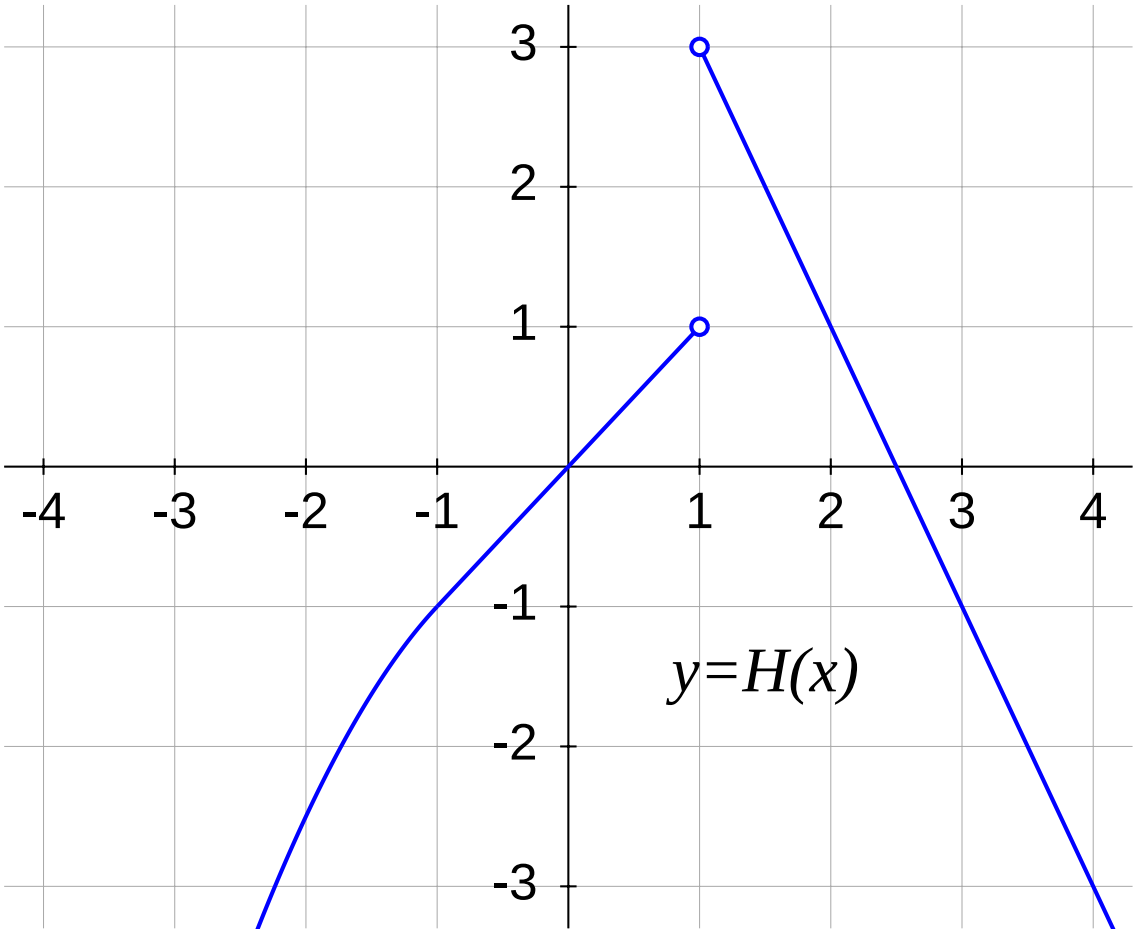
(Section L0501, November 27, 2019)

- **For next day's lecture, watch videos 6.11, 6.12.**
- Today's lecture will **assume** you have watched videos 6.3-6.10.
- Contents: Indeterminate forms and L'Hôpital's Rule.

Warm up: Limits from graphs

Compute

$$\lim_{x \to 0} \frac{H(x)}{H(2+3x)-1}$$



Proving something is an indeterminate form

1. Prove that $\forall c \in \mathbb{R}$, $\exists a \in \mathbb{R}$ and functions f and g s.t.

$$\lim_{x \to a} f(x) = 0, \quad \lim_{x \to a} g(x) = 0, \quad \lim_{x \to a} \frac{f(x)}{g(x)} = c$$

   This shows that $\frac{0}{0}$ is an indeterminate form.

2. Prove the same way that $\frac{\infty}{\infty}$, $0 \cdot \infty$, and $\infty - \infty$ are also indeterminate forms.

Computations

Calculate:

1. $\lim_{x \to 2} \dfrac{x^2 + 2x - 6}{x^2 + 3x - 10}$
2. $\lim_{x \to 0} \dfrac{e^{2x^2} - \cos x}{x \sin x}$
3. $\lim_{x \to \infty} \dfrac{x^2}{e^x}$
4. $\lim_{x \to \infty} \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$
5. $\lim_{x \to \infty} (\sin x) \left(e^{1/x} - 1 \right)$
6. $\lim_{x \to \infty} x \sin \dfrac{2}{x}$
7. $\lim_{x \to \infty} x \cos \dfrac{2}{x}$
8. $\lim_{x \to 1} \left[(\ln x) \tan \dfrac{\pi x}{2} \right]$

Careful with L'Hopital's rule

Compute

$$\lim_{x\to\infty} \frac{x+\sin x}{x}.$$

What's wrong with this argument?

INCORRECT Proof:

This is indeterminate of type $\frac{\infty}{\infty}$ so, by L'Hopital's Rule,

$$\lim_{x\to\infty} \frac{x+\sin x}{x} = \lim_{x\to\infty} \frac{1+\cos x}{1} = \lim_{x\to\infty} \big(1+\cos x\big).$$

The last limit doesn't exist so $\lim\limits_{x\to\infty} \dfrac{x+\sin x}{x}$ doesn't exist. □

Indeterminate?

Which of the following are indeterminate forms for limits?
If any of them isn't, then what is the value of such limit?

1. $\frac{0}{0}$
2. $\frac{0}{\infty}$
3. $\frac{0}{1}$
4. $\frac{\infty}{0}$
5. $\frac{\infty}{\infty}$
6. $\frac{1}{\infty}$
7. $0 \cdot \infty$
8. $\infty \cdot \infty$
9. $\sqrt{\infty}$
10. $\infty - \infty$
11. 1^{∞}
12. $1^{-\infty}$
13. 0^{0}
14. 0^{∞}
15. $0^{-\infty}$
16. ∞^{0}
17. ∞^{∞}
18. $\infty^{-\infty}$

Harder computations

Compute:

1. $\lim_{x \to \infty} \left(\frac{x+2}{x-2} \right)^{3x}$
2. $\lim_{x \to 0^+} x^x$
3. $\lim_{x \to 0} \left[\frac{\csc x}{x} - \frac{\cot x}{x} \right]$