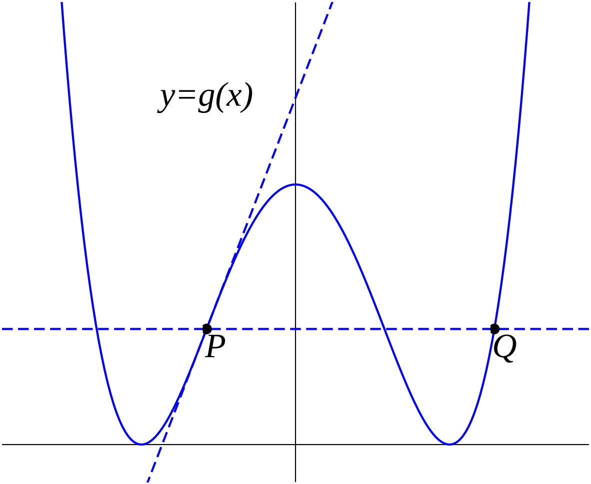
MAT137

(Section L0501, December 2, 2019)

- **For next day's lecture, watch videos 6.13, 6.14, 6.15.**
- Today's lecture will **assume** you have watched videos 6.11, 6.12.
- **Fill up the mid-year course feedback by December 13.**
- Contents: Concavity.

Warm up: Find the coordinates of P and Q

$$g(x) = x^4 - 6x^2 + 9$$



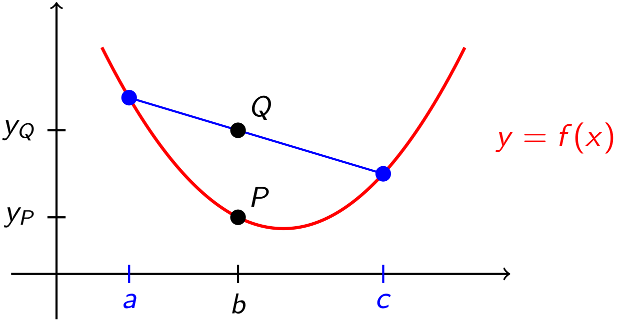
Monotonicity and concavity

Let $f(x) = xe^{-x^2/2}$.

1. Find the intervals where f is increasing or decreasing, and its local extrema.
2. Find the intervals where f is concave up or concave down, and its inflection points.
3. Calculate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
4. Using this information, sketch the graph of f.

Secants are above the graph

Let f be a function defined on an interval I. In Video 6.11 you learned that an alternative way to define "f is concave up on I" is to say that "the secant segments stay above the graph".



Rewrite this as a precise mathematical statement of the form

$$\text{"}\forall a, b, c \in I, \quad a < b < c \implies \boxed{\text{an inequality involving } f,\ a,\ b,\ c}\text{"}$$

Prove that f' is increasing on (a, c) implies the above inequality. (Use MVT)

Taylor preview

Let f be a function with domain $\mathbb{R}$.
Assume f is differentiable as many times as needed.

1. Find $a, b \in \mathbb{R}$ such that
$$\lim_{x \to 0} \frac{f(x) - [a + bx]}{x} = 0$$
2. Find $a, b, c \in \mathbb{R}$ such that
$$\lim_{x \to 0} \frac{f(x) - \left[a + bx + cx^2\right]}{x^2} = 0$$
3. Let $N \in \mathbb{N}$. Find a polynomial P_N such that
$$\lim_{x \to 0} \frac{f(x) - P_N(x)}{x^N} = 0$$

(Ans: For $0 \leq k \leq N$, the coefficient of x^k is given by $\frac{f^{(k)}(0)}{k!}$. This can be thought of as a generalization of the MVT or the L'Hopital rule, and is called Taylor's theorem.)