MAT137

(Section L0501, January 15, 2020)

- **Fot today's lecture: slides 7.10, 7.11, 7.12, 8.1, 8.2**
- **For next day's lecture, watch videos 8.3, 8.4 .**
- Contents: Riemann sums, anti derivative/indefinite integrals.
- Problem set 6 is due **Monday January 20**.

Properties of the integral

Assume we know the following

$$\int_0^2 f(x)dx = 3, \qquad \int_0^4 f(x)dx = 9, \qquad \int_0^4 g(x)dx = 2.$$

Compute:

1. $\int_0^2 f(t)dt$
2. $\int_0^2 f(t)dx$
3. $\int_2^0 f(x)dx$
4. $\int_2^4 f(x)dx$
5. $\int_{-2}^0 f(x)dx$
6. $\int_0^4 [f(x) - 2g(x)] \, dx$

Mean Value Theorem for integrals

Prove the following theorem.

Theorem

Let $a < b$. Let f be a continuous function on $[a, b]$.
There exists $c \in [a, b]$ such that

$$f(c) = \frac{1}{b-a} \int_a^b f(t)dt$$

Hints:

1. Use the upper and lower sum of the partition $\{a, b\}$ to prove that

$$??? \leq \frac{1}{b-a} \int_a^b f(t)dt \leq ???$$

2. Use the EVT.
3. Use the IVT.

Riemann sums example

Calculate $\displaystyle\int_0^1 x^2 dx$ using Riemann sums.

Hints: Imitate the calculation in Video 7.11.

1. Let $f(x) = x^2$ on $[0, 1]$.
   Let $P_n = \{\text{breaking the interval into } n \text{ equal pieces}\}$.
2. Write a explicit formula for P_n.
3. What is Δx_i?
4. Write $S^*_{P_n}(f)$ as a sum when we choose x_i^* as the right end-point.
5. Add the sum
6. Compute $\lim\limits_{n\to\infty} S^*_{P_n}(f)$.

Helpful formulas: $\displaystyle\sum_{i=1}^{N} i = \frac{N(N+1)}{2}, \qquad \sum_{i=1}^{N} i^2 = \frac{N(N+1)(2N+1)}{6}$

Riemann sums backwards

Interpret the following limits as integrals:

1. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \sin \frac{i}{n}$

2. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{n+i}{n^2}$

Warm-up questions

Let f be a continuous function defined on some interval $[a, b]$.

Question 1: The notation $\int f(x)\,dx$ represents...?

1. A number.
2. A function.
3. A collection of functions.
4. None of the above.

Question 2: What do these notations represent (same options)...?

$$\int_a^b f(x)\,dx \qquad \text{and} \qquad \int_a^x f(t)\,dt$$

True or False? $\int f(x)\,dx$ and $\int f(\mu)\,d\mu$ mean exactly the same thing.

Initial Value Problem

Find a function f such that

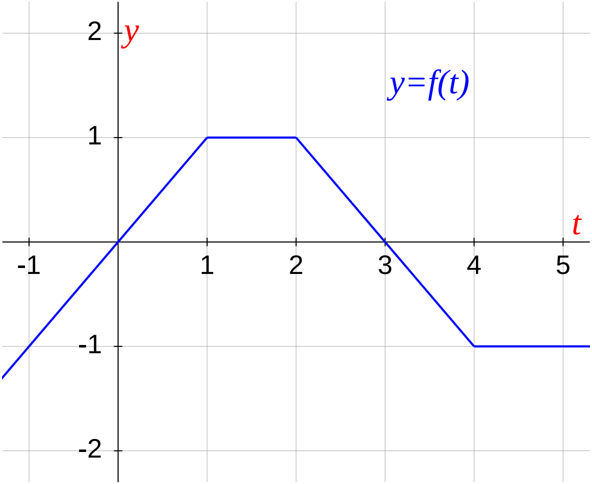
- For every $x \in \mathbb{R}$, $f''(x) = \sin x + x^2$,
- $f'(0) = 5$,
- $f(0) = 7$.

Functions defined by integrals

Which ones of these are valid ways to define functions?

1. $F(x) = \int_0^x \frac{t}{1+t^8}\, dt$
2. $F(x) = \int_0^x \frac{x}{1+x^8}\, dx$
3. $F(x) = \int_0^1 \frac{t}{1+t^8}\, dt$
4. $F(x) = \int_0^{x^2} \frac{t}{1+t^8}\, dt$
5. $F(x) = \int_{\sin x}^{e^x} \frac{t}{1+t^8}\, dt$
6. $F(x) = \int_0^3 \frac{t}{1+x^2+t^8}\, dt$

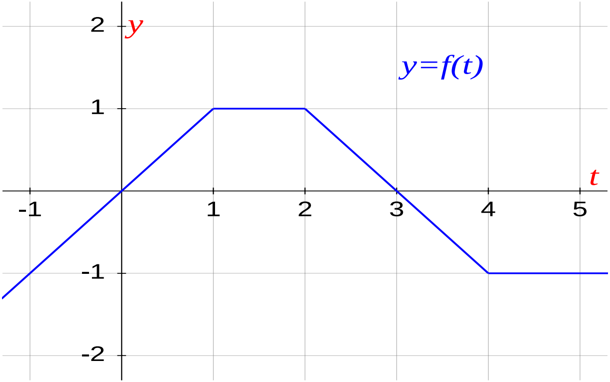
Towards FTC



Compute:

1. $\int_0^1 f(t)dt$
2. $\int_0^2 f(t)dt$
3. $\int_0^3 f(t)dt$
4. $\int_0^4 f(t)dt$
5. $\int_0^5 f(t)dt$

Towards FTC (continued)



Define $F(x) = \int_0^x f(t)dt$ for $x \in [-1, 3]$.

- Sketch the graph of $y = F(x)$.
- Using the graph you just sketched, sketch the graph of $y = F'(x)$.

Trig-exp antiderivatives

1. Calculate

$$\frac{d}{dx}\left[e^x \sin x\right], \qquad \frac{d}{dx}\left[e^x \cos x\right].$$

2. Use the previous answer to calculate

$$\int e^x \sin x \; dx, \qquad \int e^x \cos x \; dx.$$

Poly-exp antiderivatives

1. Calculate

$$\frac{d}{dx}(x^2e^{-x}) \qquad \frac{d}{dx}(xe^{-x}) \qquad \frac{d}{dx}(e^{-x}).$$

2. Use the previous answer to calculate

$$\int x^2e^{-x}\,dx$$