MAT137

(Section L0501, January 20, 2020)

- **Fot today's lecture: slides 8.3, 8.4**
- **For next day's lecture, watch videos 8.5, 8.6, 9.1–9.4 .**
- Contents: FTC 1.
- Problem set 6 is due today.

A tank is being filled with water. At time t water flows into the tank at a rate of

$$A\,e^{-bt}\arctan(ct)$$

litres per second, where A, b, and c are constants. The amount of water in the tank at time $t = 0$ s is V_0.

Write an expression for the amount of water V in the tank at time t.

True or False?

Let f and g be differentiable functions with domain $\mathbb{R}$.
Assume that $f'(x) = g(x)$ for all x.
Which of the following statements must be true?

1. $f(x) = \displaystyle\int_0^x g(t)dt$.

2. If $f(0) = 0$, then $f(x) = \displaystyle\int_0^x g(t)dt$.

3. If $g(0) = 0$, then $f(x) = \displaystyle\int_0^x g(t)dt$.

4. There exists $C \in \mathbb{R}$ such that $f(x) = C + \displaystyle\int_0^x g(t)dt$.

5. There exists $C \in \mathbb{R}$ such that $f(x) = C + \displaystyle\int_1^x g(t)dt$.

Compute the derivative of the following functions

1. $F_1(x) = \displaystyle\int_0^1 e^{-t^2}\, dt.$

2. $F_2(x) = \displaystyle\int_0^x e^{-\sin t}\, dt.$

3. $F_3(x) = \displaystyle\int_1^{x^2} \frac{\sin t}{t^2}\, dt.$

4. $F_4(x) = \displaystyle\int_x^7 \sin^3\left(\sqrt{t}\right) dt.$

5. $F_5(x) = \displaystyle\int_{2x}^{x^2} \frac{1}{1 + t^3}\, dt.$

An integral equation

Assume f is a continuous function that satisfies, for every $x \in \mathbb{R}$:

$$\int_0^x e^t f(t) dt = \frac{\sin x}{x^2 + 1}$$

Find an explicit expression for $f(x)$.

Differentiating integrals

Compute the derivative of each function:

1. $A(x) = \displaystyle\int_0^x (137 - t)^2 dt$

2. $B(x) = \displaystyle\int_0^{137} (x - t)^2 dt$

3. $C(x) = \displaystyle\int_0^x (x - t)^2 dt$

Use FTC-1 to prove for every $x > 0$ that

$$\int_0^x \frac{dt}{1+t^2} + \int_0^{1/x} \frac{dt}{1+t^2} = \frac{\pi}{2}$$