MAT137

(Section L0501, January 22, 2020)

- **Fot today's lecture: slides 8.5, 8.6, 9.1–9.4**
- **For next day's lecture, watch videos 9.5–9.9 .**
- Contents: FTC 2 and integration by substitution.

Warm up

- Define FTC1

- Define FTC2

Definite integrals

Compute

1. $\displaystyle\int_1^2 x^3\,dx$
2. $\displaystyle\int_0^1 \left[e^x + e^{-x} - \cos(2x)\right] dx$
3. $\displaystyle\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^2}}\,dx$
4. $\displaystyle\int_{\pi/4}^{\pi/3} \sec^2 x\; dx$
5. $\displaystyle\int_1^2 \left[\frac{d}{dx}\left(\frac{\sin^2 x}{1+\arctan^2 x + e^{-x^2}}\right)\right] dx$

What is wrong?

$$\int_{-1}^{1} \frac{1}{x^4}\,dx = \left.\frac{-1}{3x^3}\right|_{-1}^{1} = \frac{-2}{3}$$

However, x^4 is always positive, so the integral should be positive.

Examples of FTC-2: Areas

Calculate the area of the bounded region...

1. ... between the x-axis and $y = 4x - x^2$.
2. ... between $y = \cos x$, the x-axis, from $x = 0$ to $x = \pi$.
3. ... between $y = x^2 + 3$ and $y = 3x + 1$.
4. ... between $y = 1$, the y-axis, and $y = \ln(x + 1)$.

Symmetry

Calculate the value of these integrals **without computing any antiderivative**.

1. $\displaystyle\int_{-2}^{2} \sin x^3 \, dx$
2. $\displaystyle\int_{0}^{\pi} \cos^2 x \, dx$
3. $\displaystyle\int_{-1}^{1} \arccos x \, dx$

Hint: Sketch the graphs with Desmos. Then use symmetry to compute it. Afterwards, try to express the symmetry with an algebraic statement.

Warm up

Calculate

$$\int \frac{\sin \sqrt{x}}{\sqrt{x}}\, dx$$

Hint: Use the substitution $u = \sqrt{x}$.

Computation practice: integration by substitution

Use substitutions to compute:

1. $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

2. $\displaystyle\int e^x \cos\left(e^x\right)\,dx$

3. $\displaystyle\int \cot x\,dx$

4. $\displaystyle\int x^2\sqrt{x+1}\,dx$

Computation practice: integration by substitution

Use substitutions to compute:

1. $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$
2. $\displaystyle\int e^x \cos\left(e^x\right)\,dx$
3. $\displaystyle\int \cot x\,dx$
4. $\displaystyle\int x^2\sqrt{x+1}\,dx$
5. $\displaystyle\int \frac{e^{2x}}{\sqrt{e^x+1}}\,dx$
6. $\displaystyle\int \frac{(\ln\ln x)^2}{x\ln x}\,dx$
7. $\displaystyle\int xe^{-x^2}\,dx$
8. $\displaystyle\int e^{-x^2}\,dx$

Integral of products of sin and cos

We want to compute

$$I = \int \sin^3 x \cos^2 x \, dx$$

1. Attempt the substitution $u = \sin x$
2. Attempt the substitution $u = \cos x$
3. One worked better than the other. Which one? Why? Finish the problem.

Integral of products of sin and cos

We want to compute

$$I = \int \sin^3 x \cos^2 x \, dx$$

1. Attempt the substitution $u = \sin x$
2. Attempt the substitution $u = \cos x$
3. One worked better than the other. Which one? Why? Finish the problem.
4. Assume we want to compute

$$\int \sin^n x \cos^m x \, dx$$

When will the substitution $u = \sin x$ be helpful?
When will the substitution $u = \cos x$ be helpful?

Definite integral via substitution

This final value is right, but the write-up is WRONG. Why?

Calculate $I = \int_0^2 \sqrt{x^3+1}\, x^2 dx$

WRONG answer

Substitution: $u = x^3 + 1,\ du = 3x^2 dx.$

$$
\begin{aligned}
I = \frac{1}{3}\int_0^2 \sqrt{x^3+1}\,(3x^2 dx) &= \frac{1}{3}\int_0^2 u^{1/2}\, du \\
&= \frac{1}{3}\,\frac{2}{3}\, u^{3/2}\Big|_0^2 \\
&= \frac{1}{9}\left(x^3+1\right)^{2/3}\Big|_0^2 \\
&= \frac{2}{9}\left(2^3+1\right)^{3/2} - \frac{2}{9}(0+1)^{3/2} = \frac{52}{9}
\end{aligned}
$$