# MAT137 (Section L0501, January 29, 2020)

- Fot today's lecture: slides 9.10, 9.11, 9.12, 9.15, 9.16, 9.17
- For next day's lecture, watch videos 10.1 .
- Contents: Integration of trigonometric functions, integration of rational functions.

### Practice: Integrals with trigonometric functions

Compute the following antiderivatives. (Once you get them to a form from where it is easy to finish, you may stop.)

$$\int \sin^{10} x \cos x \, dx$$

$$\int \sin^{10} x \cos^3 x \, dx$$

$$\int e^{\cos x} \cos x \sin^5 x \, dx$$
Here are some useful trig identities:

$$\sin^2 x + \cos^2 x = 1$$
$$\tan^2 x + 1 = \sec^2 x$$

$$\int \cos^2 x \, dx$$

$$\int \sin^4 x \, dx$$

$$\int \csc x \, dx$$

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$
$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$

## A reduction formula

Let 
$$I_n = \int_0^{2\pi} \sin^n x \, dx$$
.

- **1** Compute  $I_0$  and  $I_1$ .
- Starting with  $I_n$ , use integration by parts. Then use the main trig identity to obtain an equation involving  $I_n$  and  $I_{n-2}$ .
- Use the previous answers to get a formula for  $I_n$  for every positive integer n.
- Compute  $I_8$ . (The answer should be  $\frac{35}{64}\pi$ ).

## Products of secant and tangent (Exercise)

To integrate

$$\int \sec^n x \tan^m x \, dx$$

Hint: You will need

• 
$$\frac{d}{dx} [\tan x] = \dots$$
 •  $\frac{d}{dx} [\sec x] = \dots$ 

• The trig identity involving sec and tan

# Rational integrals

$$Calculate \int \frac{1}{x+a} \, dx$$

2 Reduce to common denominator 
$$\frac{2}{x} - \frac{3}{x+3}$$

3 Calculate 
$$\int \frac{-x+6}{x^2+3x} dx$$

$$Calculate \int \frac{1}{x^2 + 3x} \, dx$$

$$\mathbf{S} \quad \mathsf{Calculate} \quad \int \frac{1}{x^3 - x} \, dx$$

#### Compute

 $\int \sec x \, dx$ 

using the substitution  $u = \sin x$ .

#### Repeated factors

## Irreducible quadratics (Solve these at home)

• Calculate 
$$\int \frac{1}{x^2 + 1} dx$$
 and  $\int \frac{x}{x^2 + 1} dx$ .

*Hint:* These two are very short. What is the derivative of arctan *x*?

2 Calculate 
$$\int \frac{2x+3}{x^2+1} dx$$

Solution Calculate 
$$\int \frac{x^2}{x^2 + 1} dx$$

• Calculate 
$$\int \frac{x}{x^2 + x + 1} dx$$

Hint: Transform it into one like the previous ones