Hamiltonian formalism for integrable systems and Riemann surfaces.

K.L. Vaninsky, MSU

The fact that Korteveg de Vriez equation and the like are Hamiltonian systems was realized in the very early days of the theory. The periodic problem for these equation is solved by methods of algebraic geometry. Novikov and Veselov in their pioneering work singled out a class of brackets for the KdV equation. They call these brackets *analytic brackets compatible with algebraic geometry.* The Gardner bracket, the Lenard-Magri bracket, bracket generated by Moser-Trubowitz isomorphism, are examples of such brackets.

In our work for basic integrable models we consider the simplest Poisson bracket which produces the flow. We introduce using the spectral transform on the auxiliary Riemann surface Γ a meromorphic function $\mathcal{X}$. We call it the Weyl function, since it is closely related to the classical Weyl function. We show that the pair $(\Gamma, \mathcal{X})$ carries a natural Poisson structure. We call it the deformed Atiyah–Hitchin bracket. It turns out that the Poisson bracket on the phase space is the image of the deformed Atiyah–Hitchin bracket under the inverse spectral transform.

This result is the first step to a systematic theory of analytic Poisson brackets compatible with algebraic geometry. We discuss open problems and state some conjectures.