MATH C34 : COMPLEX VARIABLES I
FALL 2017

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Welcome to MAT C34! In this class we will start exploring functions of one complex variable. The introduction of complex numbers has been one of the most influential innovations in mathematics, and it has made possible the development of science and technology as we know it today. In the field of complex numbers, algebra and geometry interact in an elegant and sometimes unexpected way.

This semester, we will develop the basic tools needed to work with complex numbers and holomorphic functions. The goal of this class is not just to present the facts, but also to develop your mathematical intuition and maturity. The class will be based on rigorous proofs, so you should be able to understand a proof and sometimes to create a new one. In the homework, you will be asked to answer questions which might need a certain degree of creativity.

It will be challenging but also hopefully rewarding. Good luck!

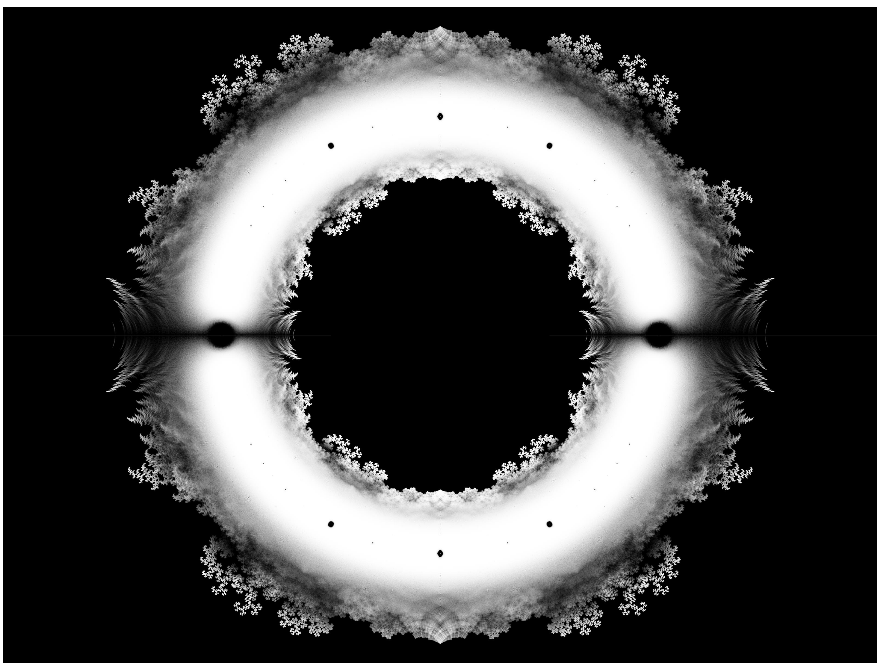

FIGURE 1. The set of complex roots of polynomials with coefficients in $\{\pm 1\}$.

SYLLABUS

PART I. INTRODUCTION TO COMPLEX NUMBERS AND FUNCTIONS

(1) The complex numbers (1.1)
(2) Convergence (1.2, 1.3)
(3) Continuous functions (2.1)
(4) Holomorphic functions (2.2)
(5) Power series (2.3)
(6) Integration along curves (3)

Part II: Cauchy's theorem and applications

(7) Goursat's theorem (1)
(8) Cauchy's theorem for a disc (2.1)
(9) Evaluation of integrals (3)
(10) Cauchy's integral formula (4)
(11) Corollaries: formula for derivatives, Cauchy inequalities
(12) Power series expansion (Theorem 4.4)
(13) Liouville's theorem and the fundamental theorem of algebra (Corollary 4.6)
(14) Morera's theorem (5.1). Sequences of holomorphic functions (5.2)
(15) Schwarz's reflection principle (5.4)

Part III: Meromorphic functions

(16) Zeros and poles (1)
(17) The residue formula (2)
(18) Applications of the residue formula (2.1)
(19) Riemann's removable singularities theorem (3.1)
(20) The Riemann sphere
(21) The argument principle (4.1)
(22) The open mapping theorem (4.4) and the maximum modulus principle (4.5)
(23) The complex logarithm (6)

Practical information

Class time.

- Tu 14:00-15:00, in SW 309
- Th 15:00-17:00, in HW 214.

Instructor. Giulio Tiozzo, tiozzo@math.toronto.edu.

Office hours. My office is IC 495. Office hours will be decided during the first few classes. Tentatively, office hours are scheduled on Tuesdays, 3-4 PM and Thursdays, 1-2 PM. If you can't make it to either, please write me and we can arrange some other time to meet.

Homework. Homework assignments will be due approximately bi-weekly at the beginning of class on Thursdays. Please staple it. Most homework will consist of writing proofs, something you may not be used to. Roughly, this means that you should always explain why your statements are true, not simply writing the final answer. Learning how to write mathematics takes some time, but it is one of the primary goals of this class.

Exams. There will be a midterm exam (on October 24^{th}, during class time) and a final written exam.

Grades. Homework 35%, midterm 25%, final 40%.

Textbooks. The main text we will use is
E. M. Stein, R. Shakarchi, *Complex analysis*
We will cover the first 3 chapters.

As a complement, other books you can have a look at are
T. Needham, *Visual complex analysis*
S. Lang, *Complex analysis*
T. Gamelin, *Complex analysis*

If you're not familiar with proofs, an interesting book to review calculus in a proof-based way is
M. Spivak, *Calculus*