(1) Prove that $H^1_{DR}(\mathbb{R}^2) = 0$

(2) Prove that an orientation ϵ on a smooth manifold M^n is continuous if and only if it's smooth, i.e. for any $p \in M$ there exists an open set $U \subset M$ containing p and a collection of **smooth** vector fields $X_1, \dots X_n$ on U such that $X_1(q), \dots X_n(q)$ is a basis of T_qM for any $q \in U$ and $\epsilon(X_1(q), \dots X_n(q)) = +1$ for any $q \in U$.

(3) Let M^n be a connected orientable manifold. Prove that there are exactly two possible orientations on M.

(4) Let $\gamma\colon [0,1] \to M^n$ be a smooth curve in M. *A smooth vector field* X *along* γ is a smooth map $X\colon [0,1] \to TM$ such that $X(t) \in T_{\gamma(t)}M$ for any t.

   (a) Suppose M^n is oriented with orientation ϵ and $X_1, \dots, X_n$ are smooth vector fields along $\gamma\colon [0,1] \to M$ such that $X_1(t), \dots, X_n(t)$ is a basis of $T_{\gamma(t)}$ for any t. Prove that $\epsilon(X_1(t), \dots, X_n(t)) = \mathit{const}$.

   (b) Prove that $\mathbb{RP}^2$ is not orientable.
   Hint: apply part (a) to $\gamma(t) = [\cos(\pi t) : \sin(\pi t) : 0]$