(1) Let M^n be a smooth manifold. Let $\pi\colon \Lambda^k(M) = A^k(M) \to M$ be the canonical projection. Let $\omega\colon M \to A^k(M)$ be a section of pi, i.e. a map satisfying $\pi \circ \omega = id_M$. Prove that the following conditions are equivalent
   (a) ω is a smooth map
   (b) in any local coordinate chart $x\colon U \to V$ where $U \subset M, V \subset \mathbb{R}^n$ are open when we write ω as $\omega = \sum_I \omega_I(x) dx^I$ then all $\omega^I(x)$ are smooth functions.
   (c) for any smooth vector fields $V_1, \dots V_k$ on M the function $f\colon M \to \mathbb{R}$ given by $f(p) = \omega(p)(V_1(p), \dots V_k(p))$ is smooth.

(2) Let $U \subset \mathbb{R}^n$ be open and let $V_1, \dots V_n$ be smooth vector fields on U such that for any $x \in U$, $V_1(x), \dots V_n(x)$ is a basis of $\mathbb{R}^n$.

   Let $w_1, \dots w_n$ be the dual collection of1 forms, i.e. for any $x \in U$ $w_1(x), \dots w_n(x)$ is the unique n-tuple of elements of $(\mathbb{R}^n)^*$ satisfying $w_i(x)(V_j(x) = \delta_{ij}$.

   Prove that all w_i are smooth.