(1) Give the following definitions
  (a) A tangent vector to a smooth manifold.
  (b) A smooth manifold with boundary.

(2) Let $f\colon \mathbb{RP}^n \to \mathbb{R}$ be given by $f([x_0 : x_1 : \ldots : x_n]) = \frac{x_n^2}{x_0^2+x_1^2+\ldots+x_n^2}$.
  (a) Show that f is well defined and smooth.
  (b) Show that the set $\{f = 1/2\}$ is nonempty and carries a natural structure of a manifold of dimension $n-1$.

(3) Mark **True or False**.
  Let M, N be smooth manifolds.
  (a) A submersion $f\colon M \to N$ is onto.
  (b) A smooth embedding $f\colon M \to N$ is a closed map.
  (c) If a smooth map $f\colon M \to N$ is injective then it's an immersion.
  (d) A local diffeomorphism which is 1-1 and onto is a diffeomorphism.
  (e) $\mathbb{S}^1$ is diffeomorphic to $\mathbb{RP}^1$.
  (f) Composition of two maps of constant rank has constant rank.

(4) Let M^n, N^m be smooth manifolds such that $n > m$. Let $f\colon M^n \to N^m$ be a smooth map.
  Prove that f is not $1-1$.
  Hint: Use the constant rank theorem.
(5) Define a map $f\colon \mathbb{S}^2 \to \mathbb{R}^4$ given by $F(x, y, z) = (x^2 - y^2, xy, xz, yz)$. Show that F induces a well defined smooth map $\bar{F}\colon \mathbb{RP}^2 \to \mathbb{R}^4$ and prove that $\bar{F}$ is an embedding.