(1) Prove that $\mathbb{RP}^n$ is Hausdorff and compact.

(2) Let $U \subset \mathbb{R}^{n+k}$ be open and let $F\colon U \to \mathbb{R}^k$ be a smooth map. Suppose $c \in \mathbb{R}^k$ is a regular value of F. It was proved in class that the level set $M = \{F = c\}$ admits a smooth atlas.

   Prove that M is Hausdorff and admits a countable smooth atlas (and hence it is a smooth manifold).

(3) Let $X = \{(x_1, \ldots, x_n) \in \mathbb{R}^n : \max_i |x_i| = 1\}$ with induced topology from $\mathbb{R}^n$. Prove that X is a topological manifold of dimension $n-1$.

(4) Let $F\colon \mathbb{R}^{n+1} \backslash \{0\} \to \mathbb{R}^2$ be given by

$$F(x_0, \ldots, x_n) = (x_0^2 + \ldots + x_n^2, \frac{x_n^2}{x_0^2 + x_1^2 + \ldots + x_n^2})$$

   Show that the set $\{F = (1, 1/2)\}$ is nonempty and carries a natural structure of a smooth manifold of dimension $n-1$.