(1) Let $U(n) = \{A \in M(n \times n, \mathbb{C})|$ such that $A \cdot A^* = Id\}$. Prove that $U(n)$ is a smooth manifold.

Hint. Consider the map $F\colon M(n \times n, \mathbb{C}) \to M(n \times n, \mathbb{C})$ given by $F(A) = A \cdot A^*$. Then $U(n) = \{F = Id\}$. Notice that $F(A)$ is always self-adjoint.

(2) Let $\pi\colon \mathbb{C}^{n+1}\backslash\{0\} \to \mathbb{CP}^n$ be the canonical projection map $\pi(z_0, \ldots, z_n) = [z_0 : \ldots : z_n]$. Let M be a smooth manifold and let $f\colon \mathbb{CP}^n \to M$ be a map.

Prove that f is smooth if and only if $f \circ \pi\colon \mathbb{C}^{n+1}\backslash\{0\} \to M$ is smooth.