(1) Prove that there exists a diffeomorphism $f\colon [0,1) \to [0,\infty)$ such that $f(x) = x$ for small x.

Hint: use partition of unity.

(2) let M be a manifold and let $\{A_i\}_{i=1}^{\infty}$ be a locally finite collection of closed subsets of M.

Prove that $A = \cup_{i=1}^{\infty} A_i$ is closed.

(3) Look at the surface of revolution M^2 in $\mathbb{R}^3$ obtained by rotating the circle of radius 1 centered at $(2,0)$ around the vertical axes.

(a) Verify that it's given by the equation

$$(\sqrt{x^2+y^2}-2)^2 + z^2 = 1$$

and that M is a smooth manifold.

(b) Prove that M is diffeomorphic to $S^1 \times S^1$.

(4) Let M, N be smooth manifolds. A map $f\colon M \to N$ is called a local diffeomorphism if for every $p \in M$ there is an open set $U \subset M$ containing p such that $f(U)$ is open in N and $f|_U\colon U \to f(U)$ is a diffeomorphism.

(a) Let G be a group acting smoothly, freely and properly discontinuosly on a smooth manifold M. Let $N = M/G$ with the induced smooth structure.

Prove that the canonical projection $\pi\colon M \to M/G$ is a local diffeomorphism

(b) Let $f\colon M \to N$ be a 1-1 local diffeomorphism where M is compact and N is connected. Prove that f is a diffeomorphism.

(c) Let $f\colon M \to N$ be a local diffeomorphism which is onto. Let P be another smooth manifold.

Prove that $g\colon N \to P$ is smooth if and only if $g \circ f$ is smooth.