(1) Let p_1, p_2 be distinct prime numbers.

Using the method from class give a careful proof of the formula

$$\phi(p_1^{k_1} p_2^{k_2}) = (p_1^{k_1} - p_1^{k_1-1})(p_2^{k_2} - p_2^{k_2-1})$$

(2) Let a, b, c be natural numbers. Let (a, b, c) be the largest natural number that divides a, b and c.
 (a) Prove that $gcd(a, b, c) = gcd(gcd(a, b), c)$.
 (b) Prove that the equation $ax + by + cz = gcd(a, b, c)$ has an integer solution.

(3) Find $22^{201} (\mod 30)$.

Note: Note that $gcd(22, 30) \neq 1$!

(4) Find $6^{3^{101}} \pmod{22}$

(5) Solve the following congruence equations
 (a) $6x \equiv 9 \pmod{33}$
 (b) $24x \equiv 7 \pmod{35}$